Sentiments, Financial Markets, and Macroeconomic Fluctuations

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March 2014

Abstract

This paper studies how financial information frictions can generate sentiment-driven fluctuations in asset prices and self-fulfilling business cycles. In our model economy, exuberant sentiments of high output and high demand for capital increase the price of capital, which signals strong fundamentals of the economy to the real side and leads to an actual boom in real output and employment. In a two-country setting, our sentiment-driven fluctuations can explain global recessions and the cross-country comovement puzzles. In the extension to the dynamic OLG setting, our model demonstrates that sentiment shocks can generate persistent fluctuations in output and employment, holding the promise to explain persistent business cycle fluctuations.

Keywords: Sentiments, Asset Prices, Business Cycles, Feedback, Real Economy

JEL codes: E2, E44, G01, G20

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1 Introduction

The financial sector plays a central role in a modern economy, which is evident from the wide and deep macroeconomic impact of the recent global financial crisis in 2007-2009. There are at least two channels through which the financial sector can influence the aggregate real economy (see, e.g., Levine (2005) and Rajan and Zingales (2004)): 1) the financing of capital, which is analogous to the role played by the "blood system" in a human body; 2) the production of information about investment opportunities, analogous to the "nervous system". An exploding financial accelerator literature pioneered by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) has shown, both theoretically and empirically, that the financial sector can influence business cycles through the *financing* channel.[1] In this paper we explore the feedback effect from financial markets to the real economy due to the *informational* role of financial prices. Unlike the conventional view that prices can efficiently allocate economic resources in a free market by signaling relevant information to economic actors (see Hayek (19945), Grossman and Stiglitz (1980)), we argue that the informational role of financial markets in allocating resources can be impaired by investors' sentiments or sunspots. The sentiment-driven asset prices in turn may influence real activities and shape macroeconomic fluctuations.

We formalize our idea in a simple benchmark three-period rational expectations model consisting of a continuum of traders and households. The traders live from period 0 to period 1. They are the initial capital owners. The households live from period 1 to period 2. The only fundamental uncertainty in the economy is the *aggregate* TFP shock in the last period (period 2). We assume, in a simple benchmark case, that only the traders have some noisy information about the TFP shock. The TFP shock in period 2 *directly* affects the return on households' investment and hence their incentive of labor supply in period 1. As capital and labor are complements in production, households' labor supply in period 1 also *indirectly* affects the traders' return on capital held from period 0 to period 1. In such an economic environment, the traders in period 0 need to forecast the level of aggregate economic activity, that is, employment and output in period 1. On the other side, forming expectations about the behavior of traders, households can obtain information about the return to their capital savings for period 2 from the price of capital in period 0. This two-way interaction between the financial market and the real economy is at the heart of our mechanism of sentiments.

Suppose that somehow exuberant sentiments lead the traders to believe there will be a boom in output in period 1. Then they conjecture that the demand for capital and therefore the return on capital will be high. Competition in the financial market will push up the capital price in period

[1] See Bernanke, Gertler and Gilchrist (1999) and a recent excellent survey by Brunnermeier, Eisenbach and Sannikov (2013).

0. However, the households cannot identify whether the high capital price is due to the traders' sentiments or their signal of a high TFP in period 2. After solving a signal extraction problem, they will attribute the high price partially to a high TFP in period 2. Their actual labor supply will indeed increase, resulting in an actual boom in output in period 1. So the traders' initial belief will be confirmed. We show that there exists a continuum of sentiment-driven equilibria, indexed by the variance of sentiments, in which the capital price reflects both sentiment and TFP shocks. Under these rational expectations equilibria, Bayesian optimal signal extraction will result in a conjectured labor supply that is always equal to actual labor supply.

We then study several generalizations of our baseline model. First, we allow more general information and signal structures, where households also receive correlated heterogeneous private signals on TFP shocks as well as market sentiments that can be correlated with the signals received by traders in period 0. We show our main results are robust to such generalizations. As a general conclusion, as long as the traders have some private information, the sentiment-driven equilibria exist. The usual argument for the efficient market hypothesis applies subtly: the market is efficient in aggregating information about TFP shocks, but it aggregates information on sentiments as well. We then also show that sentiment-driven equilibria are robust to the case in which the fundamental shocks originate from the demand side.

Then we extend our benchmark model to a two-country model, in which two symmetric countries are linked through international trade. It is well known that the standard RBC models have difficulty in explaining the comovements of output across countries (see, e.g., Perri and Quadrini (2013), Imbs (2010)) without resorting to global shocks. We show that our model is able to characterize such synchronization in the sentiment-driven equilibria. The intuition is as follows. When the economies of countries are linked through trade in intermediate goods, traders in each country need to forecast output and therefore household labor supply in both countries.

Suppose traders in both countries believe that the real economies of the two countries are highly linked through international trade (perhaps beyond the actual linkage) and a TFP shock from either country has a global impact. When traders in one country or both countries believe that there will be recession next period, and therefore they expect a low capital return, the capital prices in both countries will decline together. Next period, after observing low capital prices, households will attribute the low capital prices in part to a decline in TFP and in part to a decline in sentiments. Their low forecast of TFP will then lead them to supply less labor, save less, and induce a contraction in output. In particular, since the capital prices in both countries fall together, households will not be able to identify whether the drop in TFP or in sentiments comes from the home or the foreign country. Households in both countries will face this same "confusion". By symmetry, households in the two countries will supply similar amounts of labor, resulting in the synchronization of output, employment, and asset prices across countries. That is, perceived synchronization across countries by traders will lead to the actual synchronization.

We view our informational channel for a global recession as an addition to the existing literature on the international synchronization of business cycles during financial crises. While the theory of international synchronization proposed by Perri and Quadrini (2013) is useful to explain such synchronization among industrial countries, it alone cannot explain why many emerging countries with heavy capital controls can also suffer a deep recession. Chudik and Fratzscher (2012) find that the tightening of financial conditions is the key transmission channel only for the advanced economies in spreading the crisis. For emerging countries contagion via informational signals, driven in part by sentiments, can also be an important channel of synchronized cycles.

Finally, we extend our baseline model to a dynamic setting of an overlapping generations (OLG) model. The OLG economy is dynamically linked across periods by capital accumulation, so i.i.d. sentiment shocks can generate persistent fluctuations in output and unemployment. As persistence is a defining feature of all business cycles, this extension illustrates that sentiments also hold the promise of explaining the persistence in real data. While building a full DSGE model and confronting it with data is beyond the scope of this paper, we hope that the mechanisms developed in this paper can lay the ground for such work.

Our paper relates to several strands of literature. First, our paper adds to the growing recent literature that studies the feedback effects from financial markets to the real side of the economy due to informational frictions. A number of contributions to this literature use a partial equilibrium model to study one firm or a de-facto-one-firm aggregate economy. For example, a firm manager obtains information about the return of his own firm's investment (typically exogenously given) from financial markets. Bond, Edmans and Goldstein (2012) provide an extensive survey of this literature. By contrast, in our model with a general equilibrium framework, agents form expectations and undertake investments based on information from financial markets about the *aggregate state of the economy,* rather than about individual firms.

Our paper is closely related to Angeletos, Lorenzoni and Pavan (2010) and Goldstein, Ozdenoren and Yuan (2013). These papers also study the interaction between the real sector and the financial market. In Angeletos, Lorenzoni and Pavan (2010), information spillovers flow from the real sector to the financial sector. Entrepreneurs in the real sector base their asset investment decisions both on noisy private signals about the asset return as well as on correlated sentiments of market optimism or pessimism. Traders who then buy the assets from entrepreneurs infer the asset returns from the aggregate investment volume. The aggregate selling volume conveys information about the asset fundamentals and thus influences the asset price in the second stage, which in turn has a feedback effect on the real investment decisions and volume in the first stage. The authors then show that the correlated market sentiments can affect prices and investments by amplifying the noise in fundamentals, as well as by creating multiple market equilibria. Goldstein, Ozdenoren and Yuan (2013) study information spillovers from the financial market to the real sector,[2] as in our paper. The speculators, who have some information about the firm's investment opportunities, trade firm shares in the financial market in the first stage. The firm manager (capital provider) learns from the market price and makes his investment decision in the second stage. The authors introduce noise traders and focus on parameters that give a unique equilibrium. "Trading frenzies" can arise in their model as the firm manager optimally extracts information about investment returns from the asset price driven by the speculators' correlated signals. Both these papers illustrate mechanisms through which financial information frictions affect and amplify real activities. Allen, Morris and Shin (2006) also study market structures with sequential trading by differentially informed short-horizon traders who receive noisy public signals. They show that the public signal can indeed be over-weighted by short-term traders interested in predicting average expectations relative to the private information of final payoffs, giving rise to a Keynesian beauty contest in market prices that can systematically deviate from the average expectations of the fundamental underlying values. Finally, Benhabib and Wang (2013) construct a sequential trading model without noise traders, in which short term traders condition their trades both on private signals from fundamentals and on sunspots. Investors purchase the assets in centralized markets using market prices to form Bayesian expectations about final period returns. Benhabib and Wang (2013) show the existence of a continuum of non-fundamental sunspot equilibria. Our paper differs from Benhabib and Wang (2013) as they abstract from real economic activities and do not address feedback effects from financial markets to the real economy arising from informational frictions.

Second, our paper is related to some other recent work on self-fulfilling business cycles, which has generated renewed interest after the recent financial crisis.[3] Perri and Quadrini (2011) use self-fulfilling expectations to explain a global recession in a two-country model with financial integration. In a financially integrated economy, firms can borrow from either domestic or foreign lenders by pledging their assets. The self-fulfilling credit crunch in one country becomes global, as otherwise firms can always resort to going elsewhere for funds, making the initial belief of a credit crunch in a particular country irrational. Similarly, Bacchetta and Wincoop (2013) construct a new Keynesian two-period-two-country model with both a financial and a trade linkage. The self-fulfilling beliefs in their model rely on a real complementarity between future and current output. A belief of low second-period output leads to high precautionary saving in the first period. As consumption falls, output and profits fall and bankruptcy rates increase, leading to low realized output in the second period. With some minimum levels of financial and trade linkage, they show that a business cycle downturn will be synchronized across countries. These papers do not, however, study the

[2] Also see Goldstein, Ozdenoren and Yuan (2011).

[3] Using an approach different from sunspot multiple equilibria, Angeletos and La'O (2012) also derive sentiment-driven business cycles.

two-way interaction between financial markets and the real economy emphasized in our paper. In a closed-economy model, Benhabib, Wang and Wen (2012) study self-fulfilling business cycles in a static model with dispersed information. Production firms need to forecast both aggregate demand and their idiosyncratic demand shock from market signals about the demand. A belief of a high aggregate demand can become self-fulfilling as firms attribute high aggregate demand to their idiosyncratic demand shocks. Financial markets play no role in their model.

The paper is organized as follows. Section 2 lays out the baseline model and Section 3 presents the equilibria. Section 4 generalizes the baseline model by allowing more general information structures as well as shocks from the demand side. Section 5 studies the two-country model. Section 6 extends the model to a dynamic economy setting. Section 7 concludes.

2 The Baseline Model

We start with a three-period benchmark model with a financial sector and a real sector. The financial sector consists of a continuum of investors with unit mass. The real sector has a representative competitive firm and a continuum of households with unit mass. The investors live in periods 0 and 1 but only consume in period 1. Each investor is endowed with $K_0 = 1$ unit of capital in period 0. Investors trade capital in the financial market with price P_0 in period 0. Each unit of capital will allow its owner to receive R_1 unit of dividend (final goods) in period 1, where R_1 will be endogenized. The households live in periods 1 and 2 but only consume in period 2. The households supply labor in periods 1 and 2 to the competitive firm. The households use their wage income in period 1 to purchase final goods (saving) and become the owners of capital in period 2. The competitive firm combines capital and labor to produce a final good that can be used both for consumption and as new capital according to the production function

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{1}$$

where A_t is productivity (TFP), K_t is the firm's capital input and N_t is the firm's labor input in period $t = 1, 2$. Capital fully depreciates after production in each period.

The Firm We first describe the firm's problem. The firm solves a trivial problem. Let W_t and R_t be the real wage and the rental price (dividend) of capital, respectively. The profit maximization yields

$$\begin{aligned} W_t &= (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha}, \\ R_t &= \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha}. \end{aligned}$$

Financial Market and Information Structure The financial market opens in period 0, where the investors trade their capital among themselves based on their private information. Later when we study the OLG model, it will be clear that trading capital is equivalent to trading firm (shares) value in the financial market.[4] The only fundamental uncertainty in the economy is A_2. We assume that productivity $A_1 = 1$ and $\log A_2 = a_2$ are drawn from a normal distribution with mean $-\frac{1}{2}\sigma_a^2$ and variance σ_a^2. We assume that a_2 is realized in period 2. But the investors receive private signals about a_2 with some noise. The households do not have private information about a_2, but they can extract some information about it from the price of capital. Later we will generalize the information structure to allow a more general setting.

The investors We index investors by j. Investor j receives a private signal $s_{j0} = a_2 + \varepsilon_j$ in period 0. Investor j sells $1 - K_{j1}$ capital in period 0 and hold K_{j1} in the beginning of period 1. His consumption in period 1 is hence

$$C_{j1} = P_0(1 - K_{j1}) + R_1 K_{j1}.$$

The investor's optimal capital holdings in period 0, K_{j1}, are given by

$$\max_{K_{j1}} \mathbb{E}[\,(R_1 - P_0)\,K_{j1}|\Omega_{j0}] \tag{2}$$

where $\Omega_{j0} = \{a_2 + \varepsilon_j, P_0\}$ is the information set of investor j in period 0.

The households We index households by i. A household enjoys his consumption in period 2 and supplies labor to the firm in both periods 1 and 2. The households' utility function is given by

$$U_i = C_{i2} - \frac{\psi}{1+\gamma} N_{i1}^{1+\gamma} - \frac{\psi_2}{1+\gamma} N_{i2}^{1+\gamma}$$

Household i's budget constraints are

$$K_{i2} = W_1 N_{i1}, \tag{3}$$

$$C_{i2} = R_2 K_{i2} + W_2 N_{i2}. \tag{4}$$

For simplicity, we assume that the households supply their labor inelastically in period 2, i.e., $N_{i2} = 1$. This is automatically true if we assume $\psi_2 = 0$. Allowing an elastic labor supply in period 2 complicates the algebra but does not change the model results qualitatively. Denote $\Omega_{i1} = \{R_1, W_1, P_0\} = \Omega_1$ as the information set of household i in period 1. Using the budget

[4] In fact, our model result is exactly the same if we alternatively assume that an owner of capital is also the owner of a firm, who hires households to produce. In this case, the value of a firm is also the capital income of the firm's owner.

constraints of (3) and (4), the households' labor decision in period 1 is given by

$$\max_{N_{i1}} \mathbb{E}\left[R_2 W_1 N_{i1} - \frac{\psi}{1+\gamma} N_{i1}^{1+\gamma} | \Omega_{i1}\right]. \tag{5}$$

The first order condition of the investors' problem in (2) is

$$0 = \mathbb{E}[\,(R_1 - P_0)\,|\Omega_{j0}] \tag{6}$$

and the first order condition of the households' problem in (5) is

$$\psi N_{i1}^{\gamma} = W_1 \mathbb{E}[R_2|\Omega_{i1}]. \tag{7}$$

We also have

$$W_1 = (1-\alpha) A_1 K_1^{\alpha} N_1^{-\alpha},\; R_1 = \alpha A_1 K_1^{\alpha-1} N_1^{1-\alpha} \tag{8}$$

and

$$W_2 = (1-\alpha) A_2 K_2^{\alpha},\; R_2 = \alpha A_2 K_2^{\alpha-1} \tag{9}$$

With the above first order conditions, we are ready to define an equilibrium formally.

3 Equilibrium

Definition 1 *An equilibrium is a set of price functions $P_0 = P_0(a_2), W_1 = W_1(a_2), R_1 = R_1(a_2), W_2 = W_2(a_2), R_2 = R_2(a_2)$, and the optimal capital holdings $K_{j1} = K_1(a_2 + \varepsilon_j, P_0)$ for the investors, and $K_{i2} = K_2(R_1, W_1, P_0)$ for the households in period 2 and their labor choices $N_{i1} = N_1(R_1, W_1, P_0)$ such that: 1) Equations (6) to (9) are satisfied; 2) All markets clear*

$$\begin{aligned} \int K_{j1} dj &= 1, \\ \int N_{i1} di &= N_1 \\ \int K_{i2} di &= K_2 \end{aligned}$$

We are now ready to characterize the equilibrium. Noticing that the households are homogenous and have the same information set, i.e., $\Omega_{i1} = \Omega_1$, we therefore focus on the symmetric equilibrium where $N_{i1} = N_1$. Finding the equilibrium involves solving for the key endogenous variable N_1 from equation (7). So we first solve W_1 and R_2 and express them in terms of N_1. The following steps solve the equilibrium.

1. Given $K_1 = 1$ and N_1, we have

$$\begin{aligned} R_1 &= \alpha K_1^{\alpha-1} N_1^{1-\alpha} = \alpha N_1^{1-\alpha}, \\ W_1 &= (1-\alpha) K_1^{\alpha} N_1^{-\alpha} = (1-\alpha) N_1^{-\alpha}. \end{aligned}$$

2. The capital in period 2 is given by the labor income in period 1. Hence we have

$$K_2 = W_1 N_1 = (1-\alpha) N_1^{1-\alpha}.$$

3. We then express R_2 in term of N_1:

$$R_2 = \alpha A_2 K_2^{\alpha-1} = A_2 \alpha \left[(1-\alpha) N_1^{1-\alpha}\right]^{\alpha-1}.$$

4. In a symmetric equilibrium where $N_{i1} = N_1$, equation (7) becomes

$$\psi N_1^{\gamma} = (1-\alpha) N_1^{-\alpha} \mathbb{E}[A_2 \alpha \left[(1-\alpha) N_1^{1-\alpha}\right]^{\alpha-1} |\Omega_{i1}]$$

or

$$N_1^{\gamma+1-(1-\alpha)\alpha} = \psi^{-1} \alpha (1-\alpha)^{\alpha} \mathbb{E}\left[A_2 | \Omega_1\right].$$

5. We normalize $\psi^{-1}\alpha(1-\alpha)^{\alpha} = 1$ and denote $\theta = \frac{1}{\gamma+1-(1-\alpha)\alpha}$ and thus obtain

$$N_1 = \left\{\mathbb{E}\left[A_2|\Omega_1\right]\right\}^{\theta} = \left\{\mathbb{E}\left[A_2|P_0\right]\right\}^{\theta}. \tag{10}$$

Notice that Ω_1 is equivalent to $\{P_0\}$ as R_1 and W_1 both are a function of N_1.

6. Finally, the price P_0 should be consistent with the investors' rational expectations, namely

$$P_0 = \alpha \mathbb{E}\left[N_1^{1-\alpha}|\Omega_{j0}\right] = \alpha \mathbb{E}\left[N_1^{1-\alpha}|a_2 + \varepsilon_j, P_0\right]. \tag{11}$$

Equations (10) and (11) are the two key equations that characterize the equilibrium. Equation (10) says that the households' labor supply depends on their expectation on the real aggregate TFP shock, A_2. The financial market affects the real economy through the information channel as the households try to learn A_2 from the financial price. Equation (11) states that the price of capital depends on the marginal product of capital, which in turn depends on the real economic activities, the *aggregate* labour supply N_1. The price of capital in the financial market is higher if the investors expect an increase in the real activities. Such a two-way feedback can generate rich complementarities between the financial sector and the real sector and may result in multiple equilibria. Since solving for other variables such as W_1, K_2 and Y_2 are straightforward via steps 1-4, we will mainly focus on solving N_1 and P_0 in what follows. We will show three types of equilibria of the model.

3.1 Fully-revealing Equilibrium

We first study an equilibrium where the financial price, P_0, fully reveals the fundamental uncertainty a_2. We call this equilibrium the fully revealing rational expectations equilibrium. We have the following proposition.

Proposition 1 *There exists a fully revealing equilibrium in which*

$$\log P_0 = \log \alpha + (1-\alpha)\theta a_2, \tag{12}$$

and

$$\log N_1 = \theta a_2. \tag{13}$$

Proof. *Proof is straightforward. It is easy to see that equations (10) and (11) both hold.* ■

Equation (12) states that the capital price in period 0 efficiently aggregates all private information by noting $a_2 = \int_0^1 s_{j0} dj = \int_0^1 (a_2 + \varepsilon_j)\, dj$. Even though each investor j in period 0 gets a noisy private signal s_{j0} about a_2, which may be high or low, investors act as if they ignore their signal. In fact, if each investor believes that the dividend R_1 in the next period depends on a_2, competition in period 0 will result in that in equilibrium the price must *fully* reveal a_2. Otherwise individual investors will respond to their private information, so the information set Ω_{j0} to infer a_2 will be heterogeneous across investors, which then means that (11) cannot be true for all investors. Those who think P_0 is too low will take unbounded long positions and those who think P_0 is too high will keep selling short, so an equilibrium cannot be reached. Since the financial price fully reveals a_2, the households face no uncertainty in deciding their labor supply in period 1. As a result, their labor choice is $N_1 = \exp(\theta a_2)$. Since $R_1 = \alpha N_1^{1-\alpha}$, the capital dividend indeed depends on a_2 and it verifies the investors' initial beliefs. Hence, (12) and (13) constitute a rational expectations equilibrium.

For the fully-revealing equilibrium, the output in periods 1 and 2 is

$$\log Y_1 = (1-\alpha)\log N_1 = (1-\alpha)\,\theta a_2$$

and

$$\log Y_2 = \log\left\{A_2\left[(1-\alpha)\,Y_1\right]^{\alpha}\right\} = a_2 + \alpha\left[\log(1-\alpha) + (1-\alpha)\,\theta a_2\right],$$

respectively.

3.2 Non-revealing Equilibrium

However, there also exists a non-revealing equilibrium where the capital price does not reveal information about a_2 at all. We characterize such an equilibrium in the following proposition.

Proposition 2 *There exists a non-revealing equilibrium in which*

$$\log P_0 = \log \alpha$$

and

$$\log N_1 = 0.$$

Proof. *The proof is straightforward and hence omitted.* ■

If investors in period 0 think that the households' labor supply is $N_1 = 1$ and hence the dividend per unit capital $R_1 = \alpha N_1^{1-\alpha}$ is independent of a_2, each investor's own signal $s_{j0} = a_2 + \varepsilon_j$ then becomes irrelevant. The competition in the "efficient" financial market then drives $P_0 = R_1 = \alpha N_1^{1-\alpha} = \alpha$. Under such a price the households can learn no information about a_2 from the capital price, and thus by equation (10) their labor supply is determined by the unconditional mean of A_2, which by our assumption is one. Hence, $N_1 = 1$ or $\log N_1 = 0$. Again, the investors' initial belief that $N_1 = 1$ is verified.

For the non-revealing equilibrium, the output in periods 1 and 2 is

$$\log Y_1 = (1 - \alpha) \log N_1 = 0$$

and

$$\log Y_2 = \log \{A_2 [(1 - \alpha) Y_1]^{\alpha}\} = a_2 + \alpha \log(1 - \alpha),$$

respectively.

3.3 Sentiment-Driven Fluctuations

We now show that there are other types of equilibria in our model. We call them sentiment-driven equilibria. Suppose that the investors in the financial market also observe a non-fundamental shock, $z \sim N(0, 1)$, which is affected by their sentiment or psychology. We assume that z and a_2 are independent. That is, the information set in period 0 becomes $\Omega_{j0} = \{P_0, a_2 + \varepsilon_j, z\}$. Of course, the fully-revealing equilibrium and the non-revealing equilibrium analyzed above are still equilibria under the new information structure. We are interested in an equilibrium in which the price takes the form $\log P_0 = \bar{p} + \log\alpha + (1-\alpha)(\phi a_2 + \sigma_z z)$ and labor in period 1 is $\log N_1 = \bar{n} + \phi a_2 + \sigma_z z$, where $\bar{p}$, $\bar{n}, \phi, \sigma_z$ are coefficients to be determined. That is, in such an equilibrium sentiments matter. We have the following proposition.

Proposition 3 *There exists a continuum of equilibria indexed by $0 \leq \sigma_z^2 \leq \frac{\theta^2}{4}\sigma_a^2$, in which the price P_0 is given by*

$$\log P_0 = \bar{p} + \log\alpha + (1-\alpha)(\phi a_2 + \sigma_z z), \tag{14}$$

and

$$\log N_1 = \bar{n} + \phi a_2 + \sigma_z z \tag{15}$$

where

$$\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2\sigma_\alpha^2 - 4\sigma_z^2}}{2\sigma_\alpha}, \tag{16}$$

and $\bar{p} = \bar{n} = 0$.

Proof. *See Appendix.* ■

When investors perceive that $\log N_1 = \phi a_2 + \sigma_z z$, the dividend per unit capital, R_1, is affected by not only the fundamental shock a_2 but also by the sentiment z. Competition in the financial market in period 0 will then drive the price to $P_0 = R_1 = \alpha N_1^{1-\alpha}$. Under such a price, investors, regardless of their own signal of a_2, are happy to trade. Intuitively, again, the "efficient" market price crowds out any private learning for the investors. However, for the households, now the price P_0 can only partially reveal the fundamental shock a_2. The households face a signal extraction problem, using the price to forecast a_2. The actual labor supply will then be a function of P_0. The relative importance of the fundamental shock and the sentiment shock has to satisfy some restrictions so that the actual labor supply is exactly the same as the perceived labor supply. This explains condition (16). When condition (16) holds, the initial belief of the investors that $\log N_1 = \phi a_2 + \sigma_z z$ is verified. To see this, the actual labor supply is given by $\log N_1 = \theta\left\{\mathbb{E}\left[a_2|\phi a_2 + \sigma_z z\right] + \frac{1}{2}var(a_2|\phi a_2 + \sigma_z z)\right\}$, which can be calculated as $\log N_1 = \frac{\theta\phi\sigma_a^2}{\phi^2\sigma_a^2+\sigma_z^2}(\phi a_2 + \sigma_z z)$. We have $\frac{\theta\phi\sigma_a^2}{\phi^2\sigma_a^2+\sigma_z^2} = 1$ by rearranging (16). Therefore, P_0 and N_1 defined in equations (14) and (15) indeed constitute a rational expectations equilibrium.

For the sentiment-driven equilibria, the output in periods 1 and 2 is

$$\log Y_1 = (1-\alpha)\log N_1 = (1-\alpha)(\phi a_2 + \sigma_z z)$$

and

$$\log Y_2 = \log\left\{A_2\left[(1-\alpha)Y_1\right]^\alpha\right\} = a_2 + \alpha\left[\log(1-\alpha) + (1-\alpha)(\phi a_2 + \sigma_z z)\right],$$

respectively.

The sentiment-driven fluctuation studied in this subsection links the Keynesian notion of "beauty contests" and "animal spirits". What matters to an individual investor is not his own assessment on the fundamental a_2, but his conjecture about the actions of other investors, as in the standard beauty contest game. This comes about because of the feedback from the second stage (period 1) to the first stage (period 0), generating endogenous complementarities between actions of investors. At the same time, the sentiment shocks in the financial market affect the real economy through the price of assets and generate fluctuations in aggregate demand as if they were driven by "animal spirits".[5]

4 Discussions

4.1 More General Information Setting

We now examine the robustness of our results to alternative information structures. We show that the sunspot or sentiment-driven equilibria are robust to the generalized information structures. For expositional convenience, we repeat the equilibrium conditions of (10) and (11) here:

$$N_1 = \{\mathbb{E}\left[A_2|\Omega_{i1}\right]\}^{\theta} \tag{17}$$

$$P_0 = \alpha\mathbb{E}\left[N_1^{1-\alpha}|\Omega_{j0}\right]. \tag{18}$$

4.1.1 Heterogeneous but Correlated Sentiments

In the main setup, we assume that each investor receives a noisy signal about the fundamental a_2, that is, $s_{j0} = a_2 + \varepsilon_j$. Now we assume that the sentiment or sunspot shock z to each investor is also noisy, that is, the sentiment shock is heterogeneous but correlated across investors. Specifically, we assume that the information set in period 0 for a particular investor j is $\Omega_{j0} = \{P_0, a_2 + \varepsilon_j,\, z + \delta_j\}$ with noise on z, where $\delta_j \sim N(0, \sigma_\delta^2)$ and $cov(\varepsilon_j, \delta_j) = 0$. The information set of the households in period 1 does not change, that is, $\Omega_{j1} = \{P_0, R_1, W_1\}$.

This alternative information structure does not change the result of the sunspot or sentiment-driven equilibria in Section 4.3. In fact, under the alternative information structure, the combination of (14), (16) and (15) still satisfies conditions (17) and (18) by noting that

$$\mathbb{E}\left[A_2|P_0, a_2 + \varepsilon_j, z + \delta_j\right] = \mathbb{E}\left[A_2|P_0\right] \tag{19}$$

[5] In our model setup with asymmetric consumption periods (of investors and households), there are two frictions: the limited participation friction as in a typical OLG model and the information friction. If we focus on the second friction only, we are able to prove that the second best constrained efficiency corresponds to the fully-revealing equilibrium and the sentiment-driven equilibria are welfare reducing, which gives the welfare implication of the sentiment-driven fluctuations.

The reason for (19) is similar to that in the previous section: the "efficient" price washes out the noise of δ_j as well as ε_j. Intuitively, the price, given by (14), has already incorporated all private information and reflects a_2 and z, and hence any noises on top of a_2 and z have no value in terms of informativeness. In other words, no investor has a private information advantage when the price is a function of a_2 and z.

4.1.2 Households Having Information About Fundamentals and Sunspots

First, we assume that not only investors but also households receive signals about A_2. We show that our result of sentiment-driven equilibria is robust to this alternative information structure.

Specifically, we assume that the information set for investors in period 0 is $\Omega_{j0} = \{P_0, a_2^I + \varepsilon_j, z + \delta_j\}$ and the information set for households in period 1 is $\Omega_{i1} = \{R_1, W_1, P_0, a_2^H + v_i\}$, where $s_{j0} = a_2^I + \varepsilon_j$ is the private signal about a_2 received by investor j in period 0 and $s_{i1} = a_2^H + v_i$ is the private signal about a_2 of household i in period 1. We assume that $cov(a_2, a_2^I) > 0$, $cov(a_2, a_2^H) > 0$ and $cov(a_2^I, a_2^H) = 0$. For instance, $a_2 = \omega a_2^I + (1 - \omega)\, a_2^H$, with $0 < \omega < 1$ and $cov(a_2^I, a_2^H) = 0$, satisfies the assumptions. Without loss of generality, we assume that $a_2 = a_2^I + a_2^H$. In addition, we assume the unconditional distributions that $a_2^I \sim N(-\frac{1}{2}\sigma_I^2, \sigma_I^2)$ and $a_2^H \sim N(-\frac{1}{2}\sigma_H^2, \sigma_H^2)$.

Under the above alternative information structure, we have the following proposition.

Proposition 4 *There exists a continuum of sunspot equilibria indexed by $0 \leq \sigma_z^2 \leq \frac{\theta^2}{4}\sigma_I^2$, in which the price P_0 is given by*

$$\log P_0 = \log \alpha + (1 - \alpha)\left(\phi a_2^I + \sigma_z z\right), \tag{20}$$

and

$$\log W_1 = \log(1 - \alpha) - \alpha\left[\left(\phi a_2^I + \sigma_z z\right) + \theta a_2^H\right], \tag{21}$$

$$\log N_1 = \left(\phi a_2^I + \sigma_z z\right) + \theta a_2^H, \tag{22}$$

where

$$\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2\sigma_I^2 - 4\sigma_z^2}}{2\sigma_I}, \tag{23}$$

Proof. *See Appendix.* ■

The intuition behind Proposition 4 is similar to that of Proposition 3. When households decide their labor supply, they need to forecast $a_2 = a_2^I + a_2^H$. They can infer a_2 through three pieces of information: financial price P_0, wage W_1 and their own signal $s_{i1} = a_2^H + v_i$. Wage W_1 efficiently aggregates all private signals, s_{i1}, to clear the labor market. This can be understood by noting that the total labor demand is $N_1^d = \left(\frac{W_1}{1-\alpha}\right)^{-\frac{1}{\alpha}}$, which only depends on the wage. The household labor supply is characterized by some function N_1^s such that $N_{i1}^s = N_1^s(W_1, a_2^H + v_i, P_0)$. The market clearing condition requires $N_1^d = \int N_1^s(W_1, a_2^H + v_i, P_0)di$, which means $W_1 = W(P_0, a_2^H)$ for any function N_1^s. Since households knows P_0, they can infer a_2^H perfectly from W_1.

We can further generalize the information structure by allowing investors' and households' signals on A_2 to be correlated. In addition, we can allow households to also receive some information about sunspots and their signals on sunspots to be correlated with investors'. Specifically we assume that $a_2 = a_2^I + d + a_2^H$, where d is a random variable independent of a_2 and $d \sim N(-\frac{1}{2}\sigma_d^2, \sigma_d^2)$; a_2^I and a_2^H have the unconditional distributions as previously specified with $cov(a_2^I, a_2^H) = 0$. Similarly, we assume that $z = z^I + \chi + z^H$, where z^I, χ and z^H all have the standard normal unconditional distribution, and $cov(z, \chi) = 0$ and $cov(z^I, z^H) = 0$. The information set for investors is assumed to be $\Omega_{j0} = \{P_0, a_2^I + d + \varepsilon_j,\ z^I + \chi + \delta_j, d, \chi\}$ and the information set for households is $\Omega_{i1} = \{R_1, W_1, P_0, a_2^H + d + v_i, z^H + \chi + \varsigma_i, d, \chi\}$, where $\varsigma_i \sim N(0, \sigma_\varsigma^2)$. In other words, the signals on both a_2 and z are correlated across investors and households, where d and χ represent common information for investors and households. Under this alternative information structure, we show that there exists an continuum of sunspot equilibria indexed by $0 \le \sigma_z^2 \le \frac{\theta^2}{4}\sigma_I^2$, with

$$\log P_0 = \log\alpha + (1-\alpha)\left(\phi a_2^I + \sigma_z z^I + \theta d\right),$$

$$\log W_1 = \log(1-\alpha) - \alpha\left[\left(\phi a_2^I + \sigma_z z^I + \theta d\right) + \theta a_2^H\right],$$

$$\log N_1 = \left(\phi a_2^I + \sigma_z z^I + \theta d\right) + \theta a_2^H,$$

where ϕ is given by

$$\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2\sigma_I^2 - 4\sigma_z^2}}{2\sigma_I}.$$

The proof is very similar to that of Proposition 4 and hence is omitted. A conclusion we can draw is that as long as the investors have some private information there exist sunspot equilibria, even if the households receive sunspot and fundamental signals correlated with the private signals of the investors. So without loss of generality, in the following sections, we assume that only investors have information about the fundamental shock A_2 and the sunspot z.

4.2 Demand Shocks

In this subsection, we show that sentiment-driven equilibria are robust to the case in which the fundamental shocks come from the demand side. We slightly change the setup in the benchmark model. Specifically, we assume that there is no uncertainty on A_2 and $A_2 = 1$, and instead there is a preference shock for households in period 2 whose utility function is given by

$$U_i = \exp(\xi) \cdot C_{i2} - \frac{\psi}{1+\gamma} N_{i1}^{1+\gamma}$$

where ξ is a random variable with distribution $\xi \sim N(-\frac{1}{2}\sigma_\xi^2, \sigma_\xi^2)$. Likewise, the investors' utility changes to $U_j = \exp(\xi) \cdot C_{j1}$. Investor j solves

$$\max_{K_{j1}} \mathbb{E}[\exp(\xi) \cdot [P_0(1-K_{j1}) + R_1 K_{j1}] \, |\Omega_{j0}]$$

As in the baseline model, we assume that only investors have private information regard ξ. In particular, investor j receives a private signal $s_{j0} = \xi + \varepsilon_j$ in period 0, that is, the information set of investor j in period 0 is $\Omega_{j0} = \{\xi + \varepsilon_j, P_0\}$. The information set in period 1 does not change, i.e., $\Omega_{i1} = \{R_1, W_1, P_0\} = \Omega_1$. In this case, the households' labor decision in period 1 is given by

$$\max_{N_{i1}} \mathbb{E}\left[\exp(\xi) \cdot R_2 W_1 N_{i1} - \frac{\psi}{1+\gamma} N_{i1}^{1+\gamma} |\Omega_{i1}\right].$$

Equilibrium conditions (10) and (11) are respectively replaced by

$$N_1 = \{\mathbb{E}[\exp(\xi)\,|\Omega_1]\}^\theta = \{\mathbb{E}[\exp(\xi)\,|P_0]\}^\theta$$

and

$$P_0 = \alpha \frac{\mathbb{E}\left[\exp(\xi) N_1^{1-\alpha} |\Omega_{j0}\right]}{\mathbb{E}[\exp(\xi)\,|\Omega_{j0}]} = \alpha \mathbb{E}\left[N_1^{1-\alpha} |\xi + \varepsilon_j, P_0\right],$$

where the second equality follows by the law of iterated expectations.

Under this alternative setup, it is easy to show that there exists a continuum of equilibria indexed by $0 \le \sigma_z^2 \le \frac{\theta^2}{4}\sigma_\xi^2$, in which the price P_0 is given by

$$\log P_0 = \log \alpha + (1-\alpha)(\phi\xi + \sigma_z z),$$

and

$$\log N_1 = \phi\xi + \sigma_z z,$$

where $\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2\sigma_\xi^2 - 4\sigma_z^2}}{2\sigma_\xi}$. This equilibrium is identical to the equilibrium in Proposition 3; the only difference is that a_2 is replaced by ξ and σ_a^2 is replaced by σ_ξ^2.

5 A Two-Country Model

We now extend our baseline model to two countries to study the international comovement. Output tends to be highly correlated across the U.S. and the remaining major industrialized countries. Yet standard international real business cycle models have great difficulty in explaining this fact (see the survey on international RBC models by Backus et al. (1995) and Baxter (1995)). A prominent feature of the great recession is that it is global. Perri and Quadrini (2013) documented that all major industrialized countries experienced extraordinarily large and unprecedentedly synchronized contractions in output and asset prices. Explaining such an unprecedented degree of international synchronization is difficult for the standard international RBC models without resorting to some global shocks. The purpose of the two-country extension here is to show that our sentiment-driven equilibrium can generate fully synchronized fluctuations in unemployment and asset prices even though each country faces its own productivity and sentiment shocks.

Our model economy consists of two symmetric countries $\ell = h$ and f, linked by international trade. In each country, the final goods in home country is produced by

$$Y_t = \left(\frac{X_{ht}}{\eta}\right)^{\eta} \left(\frac{X_{ht}^*}{1-\eta}\right)^{1-\eta}, \tag{24}$$

where X_{ht}^* is the imported material (intermediate) goods from the foreign country. So each intermediate goods firm in the two countries serves the markets of both countries: providing material goods to the domestic final goods firms as well as the foreign final goods firms.

The investors only consume the domestic final goods in period 1. The households only consume the domestic final goods in period 2 and their utility function is given by

$$U_i = C_{i2} - \frac{\psi}{1+\gamma} N_{i1}^{1+\gamma},$$

where C_{i2} is the consumption on the final goods in period 2.

We normalize the final goods price to be 1 in each country and let e_t denote the price of foreign final goods in terms of home final goods (the real exchange rate). Taking advantage of the symmetry, we just need to describe the home country. We assume that $\eta > \frac{1}{2}$ to represent the well-known home bias in international trade.

The representative final goods firm in the home country solves

$$\max_{X_{ht}, X_{ht}^*} \left(\frac{X_{ht}}{\eta}\right)^{\eta} \left(\frac{X_{ht}^*}{1-\eta}\right)^{1-\eta} - P_{ht} X_{ht} - e_t P_{ft} X_{ht}^*. \tag{25}$$

where P_{ht} is the price of the home intermediate goods in period t in the home country and P_{ft} is the price of the foreign intermediate goods in period t in the foreign country. Because of the constant-returns-to-scale production function, the firm makes zero profit and thus

$$Y_t \cdot 1 = P_{ht} X_{ht} + e_t P_{ft} X_{ht}^*$$

By the property of the Cobb–Douglas production function, we have

$$P_{ht}X_{ht} = \eta Y_t, \tag{26}$$

$$e_t P_{ft} X^*_{ht} = (1-\eta) Y_t. \tag{27}$$

Let X_t be the total production of intermediate goods in the home country in period t. The intermediate goods is produced by competitive firms using domestic capital and labor according to the production function

$$X_t = A_t K_t^{\alpha} N_t^{1-\alpha}. \tag{28}$$

The first order conditions of (28) with respect to K_t and N_t are

$$\begin{aligned} R_t &= \alpha P_{ht} A_t K_t^{\alpha-1} N_t^{1-\alpha}, \\ W_t &= (1-\alpha) P_{ht} A_t K_t^{\alpha} N_t^{-\alpha} \end{aligned} \tag{29}$$

respectively.

Since there are no financial assets traded between the two countries, trade has to be balanced. The home country exports total $X_t - X_{ht}$ home intermediate goods and imports X^*_{ht} foreign intermediate goods, so balanced trade leads to

$$P_{ht}(X_t - X_{ht}) = e_t P_{ft} X^*_{ht}, \tag{30}$$

or

$$P_{ht} X_t = P_{ht} X_{ht} + e_t P_{ft} X^*_{ht} = Y_t \tag{31}$$

The second equality in (31) follows (26) and (27). Combining (31) and (26) yields

$$X_{ht} = \eta X_t. \tag{32}$$

Exploiting the symmetry between the two countries, we also obtain

$$X^*_{ht} = (1-\eta) X^*_t. \tag{33}$$

Using (31) and substituting (32) and (33) into (24), we have

$$P_{ht} = \frac{Y_t}{X_t} = \left(\frac{X^*_t}{X_t}\right)^{1-\eta}$$

Using (27) and substituting with (33) and $P_{ft}X_t^* = Y_t^*$ also yield

$$e_t = \frac{(1-\eta)Y_t}{P_{ft}X_{ht}^*} = \frac{(1-\eta)Y_t}{(1-\eta)Y_t^*} = \frac{Y_t}{Y_t^*}$$

Finally, by substituting (32), (33) and (28) into (24), we obtain

$$\begin{aligned} Y_t &= \left(\frac{X_{ht}}{\eta}\right)^\eta \left(\frac{X_{ht}^*}{1-\eta}\right)^{1-\eta} = X_t^\eta X_t^{*1-\eta} \\ &= \left(A_t K_t^\alpha N_t^{1-\alpha}\right)^\eta \left(A_t^* K_t^{*\alpha} N_t^{*1-\alpha}\right)^{1-\eta} \end{aligned} \tag{34}$$

As in the benchmark model, we assume that $A_1 = 1$ and $A_1^* = 1$; the only fundamental uncertainty in the economy is productivity level in period 2, namely A_2 and A_2^*.[6]

The investors in period 0 trade capital to solve

$$\max_{K_{j1}} \mathbb{E}\left[P_0 + (R_1 - P_0)K_{j1} | P_0, a_2 + \varepsilon_j, P_0^*, z\right], \tag{35}$$

where P_0 is the capital price in home country and P_0^* is the capital price in the foreign country. We assume that investors also receive a private signal regarding domestic productivity a_2 plus a sentiment shock z in the home country. That is, $\Omega_1 = \{P_0, a_2 + \varepsilon_j, P_0^*, z\}$ and $\Omega_1^* = \{P_0^*, a_2^* + \varepsilon_j^*, P_0, z^*\}$. Equation (35) yields

$$P_0 = \mathbb{E}\left[R_1 | P_0, a_2 + \varepsilon_j, P_0^*, z\right],$$

and by symmetry

$$P_0^* = \mathbb{E}\left[R_1^* | P_0^*, a_2^* + \varepsilon_j^*, P_0, z^*\right].$$

The labor decision in period 1 is given by

$$\psi N_1^\gamma = W_1 \mathbb{E}[R_2 | \Omega_1] \tag{36}$$

$$\psi N_1^{*\gamma} = W_1^* \mathbb{E}[R_2^* | \Omega_1^*], \tag{37}$$

where $\Omega_1 = \{P_0, W_1, R_1, P_0^*\}$, and $\Omega_1^* = \{P_0^*, W_1^*, R_1^*, P_0\}$. The budget constraints of the households imply

$$K_2 = W_1 N_1 = (1-\alpha)Y_1 \tag{38}$$

$$K_2^* = W_1^* N_1^* = (1-\alpha)Y_1^*. \tag{39}$$

[6] Also, we assume that the households supply their labor inelastically in period 2, that is, $N_2 = 1$ and $N_2^* = 1$, as in the baseline model.

To characterize all equilibrium variables, we will determine N_1 and N_1^* first. Normalize $\psi^{-1}\alpha(1-\alpha)^{\alpha}=1$ as in the baseline model in (10) and let $\lambda=\frac{1}{\gamma+1-\alpha(1-\alpha)(2\eta^2-2\eta+1)}$ and $\omega=\lambda 2\alpha\eta\,(1-\alpha)\,(1-\eta)$. We have the following proposition regarding equilibrium N_1 and N_1^*.

Proposition 5 *Equilibrium N_1 and N_1^* jointly satisfy*

$$N_1 = N_1^{*\omega}\left\{\mathbb{E}[A_2^{\eta}A_2^{*1-\eta}|P_0, W_1, R_1, P_0^*]\right\}^{\lambda}, \tag{40}$$

$$N_1^* = N_1^{\omega}\left\{\mathbb{E}[A_2^{*\eta}A_2^{1-\eta}|P_0^*, W_1^*, R_1^*, P_0]\right\}^{\lambda}, \tag{41}$$

where P_0 and P_0^* are given by

$$P_0 = \alpha\mathbb{E}\left[\left(N_1^{1-\alpha}\right)^{\eta}\left(N_1^{*1-\alpha}\right)^{1-\eta}|P_0, a_2+\varepsilon_j, P_0^*, z\right], \tag{42}$$

$$P_0^* = \alpha\mathbb{E}\left[\left(N_1^{*1-\alpha}\right)^{\eta}\left(N_1^{1-\alpha}\right)^{1-\eta}|P_0^*, a_2^*+\varepsilon_j^*, P_0, z^*\right], \tag{43}$$

respectively.

Proof. *See Appendix.* ■

There exist three types of equilibria in the model as in the baseline case.

5.1 Fully-revealing Equilibrium

In the fully-revealing equilibrium, sentiments do not matter. In addition, the households can perfectly infer two fundamental shocks after observing two asset prices: P_0 and P_0^* . Then households in both countries face no uncertainty in deciding their labor supply. The following proposition characterizes the fully-revealing equilibrium.

Proposition 6 *There exists a fully-revealing equilibrium in which the capital prices are given by*

$$\begin{aligned} p_0 &= \log P_0 = \log\alpha + \pi_h a_2 + \pi_f a_2^*, \\ p_0^* &= \log P_0^* = \log\alpha + \pi_h a_2^* + \pi_f a_2 \end{aligned}$$

and the labor supplies are

$$\begin{aligned} n_1 &= \log N_1 = \theta_h a_2 + \theta_f a_2^*, \\ n_1^* &= \log N_1^* = \theta_h a_2^* + \theta_f a_2. \end{aligned}$$

where $\pi_h=\frac{\lambda(1-\alpha)}{1-\omega^2}\left[2\,(1-\eta)\,\eta\omega+\eta^2+(1-\eta)^2\right]$, $\pi_f=\frac{\lambda(1-\alpha)}{1-\omega^2}\left[\left(\eta^2+(1-\eta)^2\right)\omega+2\,(1-\eta)\,\eta\right]$, $\theta_h=\frac{\lambda}{1-\omega^2}\left[(1-\eta)\,\omega+\eta\right]$ *and* $\theta_f=\frac{\lambda}{1-\omega^2}\left[\eta\omega+(1-\eta)\right]$.

Proof. *The proof is straightforward.* ■

Notice that as long as $\eta \neq \frac{1}{2}$, we have that $\pi_h - \pi_f = \frac{\lambda(1-\alpha)}{1+\omega}(1-2\eta)^2 \neq 0$. So households can perfectly infer a_2 and a_2^* from prices. Formally, $a_2 = \frac{\pi_h(p_0 - \log\alpha) - \pi_f(p_0^* - \log\alpha)}{\pi_h^2 - \pi_f^2}$ and $a_2^* = \frac{\pi_h(p_0^* - \log\alpha) - \pi_f(p_0 - \log\alpha)}{\pi_h^2 - \pi_f^2}$.

In this fully-revealing equilibrium, the output in the two countries in period 1 is given by

$$\begin{aligned} y_1 &= \log Y_1 = (1-\alpha)\left[\eta n_1 + (1-\eta) n_1^*\right] \\ y_1^* &= \log Y_1^* = (1-\alpha)\left[\eta n_1^* + (1-\eta) n_1\right] \end{aligned}$$

Due to international trade, the output in the first period is correlated even if $corr(a_2, a_2^*) = 0$. However, the comovement is weak when η is close to 1. In the limiting case

$$\lim_{\eta\to 1} cov(y_1, y_1^*) = 0,$$

by noting

$$\lim_{\eta\to 1} \omega \to 0, \lim_{\eta\to 1} \theta_h = \lim_{\eta\to 1} \lambda = \theta, \text{ and } \lim_{\eta\to 1} \theta_f \to 0,$$

where θ is defined in (10). Since matching the small bilateral trade to GDP ratio means a large η, explaining international comovements is difficult.

Once we obtain n_1 and n_1^*, it is straightforward to derive other variables. In period 1,

$$\log P_{h1} = (1-\eta)(1-\alpha)(n_1^* - n_1)$$

and the real exchange rate is

$$\log e_1 = y_1 - y_1^*.$$

In period 2, the output is

$$\begin{aligned} y_2 &= \log Y_2 = \alpha\log(1-\alpha) + \left[\eta a_2 + (1-\eta) a_2^*\right] + \alpha\left[\eta y_1 + (1-\eta) y_1^*\right] \\ y_2^* &= \log Y_2^* = \alpha\log(1-\alpha) + \left[\eta a_2^* + (1-\eta) a_2\right] + \alpha\left[\eta y_1^* + (1-\eta) y_1\right]. \end{aligned}$$

5.2 Non-revealing Equilibrium

In the non-revealing equilibrium, the prices of capital in the two countries are

$$\begin{aligned} \log P_0 &= \log P_0^* = \log\left[\alpha\left\{\mathbb{E}[A_2^{*\eta} A_2^{1-\eta}]\right\}^{\frac{\lambda(1-\alpha)}{1-\omega}}\right] \\ &= \log\alpha + \frac{\lambda(1-\alpha)}{1-\omega}\eta(\eta-1)\sigma_\alpha^2 \end{aligned}$$

and the labor supplies are

$$\begin{aligned}\log N_1 &= \log N_1^* = \frac{\lambda}{1-\omega}\log\left[\mathbb{E}\left(A_2^{\eta}A_2^{*1-\eta}\right)\right]\\ &= \frac{\lambda}{1-\omega}\eta(\eta-1)\sigma_\alpha^2.\end{aligned}$$

In addition, we have that $P_{h1}=1$, $e_1=1$ and

$$\log Y_1 = \log Y_1^* = (1-\alpha)\frac{\lambda}{1-\omega}\eta(\eta-1)\sigma_\alpha^2$$

and

$$\begin{aligned}\log Y_2 &= \alpha\log(1-\alpha)+\left[\eta a_2+(1-\eta)a_2^*\right]+\alpha\left[(1-\alpha)\frac{\lambda}{1-\omega}\eta(\eta-1)\sigma_\alpha^2\right]\\ \log Y_2^* &= \alpha\log(1-\alpha)+\left[\eta a_2^*+(1-\eta)a_2\right]+\alpha\left[(1-\alpha)\frac{\lambda}{1-\omega}\eta(\eta-1)\sigma_\alpha^2\right].\end{aligned}$$

The intuition for the non-revealing equilibrium is similar to the previous closed-economy model. If investors believe that the labor supply of households in the two countries in period 1 is the same and independent of fundamental shocks and sentiments, they will believe that dividend per unit of capital, R_1 and R_1^*, in the two countries also becomes the same and independent of fundamental shocks and sentiments. As a result, competition in the financial market will drive the capital prices to $P_0 = R_1 = R_1^* = P_0^*$. This, in turn, means that households in both countries will have no information about sentiments and productivity shocks. They will supply their labor according to the unconditional mean of A_2 and A_2^*. The initial beliefs of investors are hence self-fulfilling.

5.3 Sentiment-Driven Comovement

We now consider the sentiment-driven equilibrium. Parallel to the case in the closed economy, we have the following proposition.

Proposition 7 *There exists a continuum of equilibria indexed by* $0 \le \sigma_z^2 \le \frac{\lambda^2\sigma_a^2}{16(1-\omega)^2}$*, in which*

$$\log P_0 = \log P_0^* = \overline{\overline{p}} + (1-\alpha)\left[\phi(a_2+a_2^*)+\sigma_z(z+z^*)\right] \tag{44}$$

$$\log N_1 = \log N_1^* = \overline{\overline{n}} + \phi(a_2+a_2^*)+\sigma_z(z+z^*) \tag{45}$$

$$\log Y_1 = \log Y_1^* = (1-\alpha)\left[\phi(a_2+a_2^*)+\sigma_z(z+z^*)\right]$$

where

$$\phi = \frac{\lambda\sigma_a^2 \pm \sqrt{\left(\lambda\sigma_a^2\right)^2 - 16(1-\omega)^2\sigma_a^2\sigma_z^2}}{4(1-\omega)\sigma_a^2} \tag{46}$$

and

$$\begin{aligned} \overline{\overline{n}} &= \frac{\lambda}{2(1-\omega)} \left\{ \left[\eta^2 + (1-\eta)^2 - 1\right] + \frac{\phi^2\sigma_a^2}{2\phi^2\sigma_a^2 + 2\sigma_z^2} \right\} \sigma_a^2 \\ \overline{\overline{p}} &= \log\alpha + (1-\alpha)\,\overline{\overline{n}}. \end{aligned}$$

That is, output as well as employment in period 1 in the two countries becomes fully synchronized.

Proof. *See appendix.* ■

The intuition behind the above proposition is as follows. Suppose the investors in each country believe that the labor supply of the households is driven by global factors such as $a_2 + a_2^*$ and $z + z^*$. Investors believe that the real economies in the two countries are fully synchronized and a shock from one country has an *equal* impact on the home country and the foreign country. Thus, they conjecture that $R_1 = \alpha N_1^{1-\alpha} = \alpha N_1^{*1-\alpha} = R_1^*$. Competition in the market will then drive $P_0 = R_1$ and $P_0^* = R_1^*$. In this case, asset prices are fully synchronized. The households, on the other hand, rationally understand that a_2 and a_2^* have a *different* impacts in their own country (as long as $\eta \neq \frac{1}{2}$). They need to solve a signal extraction problem by inferring a_2 and a_2^* from the common price $P_0 = P_0^*$. However, because $P_0 = P_0^*$, households cannot distinguish the home TFP shock from the foreign TFP shock from observing asset prices. Households in both countries face the same confusion, so by symmetry, households in the two countries provide the same level of labor. This means a perfectly synchronized labor supply across countries. So investors' initial beliefs of perfect synchronization are verified. Of course, for the actual labor supply to be equal to the conjectured labor supply defined in equation (45), the relative importance of fundamental shocks and sentiments has to satisfy a restriction given by (46).

The extreme case of $\eta = 1$ helps further highlight the intuition. Note that $\eta = 1$ means that there is no international trade linkage between the two countries. However, the perfect synchronization of sentiment-driven equilibria in Proposition 7 still holds. Intuitively, investors in each country "overreact" to the foreign shock. Although a_2^* has no impact on the domestic economy at all, the investors in the home country think it does, and in trading their capital they react to the foreign asset price $\phi a_2^* + \sigma_z z^*$. In this case, $\phi a_2^* + \sigma_z z^*$ essentially becomes another sunspot in addition to z, that is, there are two sunspot shocks for the investors in the home country. By symmetry, if investors in the foreign country also "overreact" to $\phi a_2 + \sigma_z z$, the asset prices in the two countries can become perfectly synchronized. The "overreactions" of investors in the two countries have real impacts, and result in self-fulfilling equilibria. In this sense, sentiments in financial markets can *amplify* the cross-country comovement.

For the sentiment-driven equilibria, the output in period 2 is given by

$$\log Y_2 = \alpha \log(1-\alpha) + [\eta a_2 + (1-\eta)\, a_2^*] + \alpha \log Y_1 \tag{47}$$
$$\log Y_2^* = \alpha \log(1-\alpha) + [\eta a_2^* + (1-\eta)\, a_2] + \alpha \log Y_1^*. \tag{48}$$

Because $cov(y_1, y_1^*) = 1$, we have that $cov(y_2, y_2^*)$ is higher in the sentiment-driven equilibria than in the fully-revealing equilibrium, for any given η and $corr(a_2, a_2^*)$. That is, the sentiment-driven equilibria result in a stronger cross-country correlation in output in period 2, beyond what can be explained by the correlation of TFPs in period 2.

It is easy to extend the information structure to allow imperfect comovement in period 1 for the sentiment-driven equilibrium. The intuition is as follows. In the fully-revealing equilibrium the comovement between two countries in period 1 is not perfect (as long as $\eta \neq \frac{1}{2}$), while in the sentiment-driven equilibria the comovement can be perfect. Thus, we can construct an information structure to make an equilibrium that is a mix of the two equilibria above. Specifically, we can assume that the TPF in period 2 in each country has two components: the observed component and the unobserved component, that is, the log TFP in the home country is $a_2 + a_2^O$ and in the foreign country is $a_2^* + a_2^{*O}$. The information structure is $\Omega_0 = \left\{P_0, a_2 + \varepsilon_j, P_0^*, a_2^O, a_2^{*O}, z\right\}$, $\Omega_0^* = \left\{P_0^*, a_2^* + \varepsilon_j^*, P_0, a_2^O, a_2^{*O}, z^*\right\}$, $\Omega_1 = \left\{P_0, W_1, R_1, P_0^*, a_2^O, a_2^{*O}\right\}$ and $\Omega_1 = \left\{P_0^*, W_1^*, R_1^*, P_0, a_2^O, a_2^{*O}\right\}$, which means that a_2^O and a_2^{*O} are public information for both countries from period 0. We also assume that $cov(a_2^O, a_2^{*O}) = 0$. Hence, the TFPs in the two countries in period 2 are not perfectly correlated. We can find sentiment-driven equilibria with asset prices and labor supplies respectively as

$$p_0 = \log P_0 = \overline{\overline{p}} + (1-\alpha)\left[\phi(a_2 + a_2^*) + \sigma_z(z + z^*)\right] + \pi_h a_2^O + \pi_f a_2^{O*}$$
$$p_0^* = \log P_0^* = \overline{\overline{p}} + (1-\alpha)\left[\phi(a_2 + a_2^*) + \sigma_z(z + z^*)\right] + \pi_h a_2^{O*} + \pi_f a_2^O$$

and

$$\log N_1 = \overline{\overline{n}} + \left[\phi(a_2 + a_2^*) + \sigma_z(z + z^*)\right] + \theta_h a_2^O + \theta_f a_2^{O*}$$
$$\log N_1^* = \overline{\overline{n}} + \left[\phi(a_2 + a_2^*) + \sigma_z(z + z^*)\right] + \theta_h a_2^{O*} + \theta_f a_2^O,$$

where $\overline{\overline{p}}$, ϕ, σ_z, π_h, π_f, $\overline{\overline{n}}$, θ_h are θ_f are coefficients to be solved. In such equilibria, the output in period 1 is

$$\log Y_1 = (1-\alpha)\left[\eta \log N_1 + (1-\eta)\log N_1^*\right]$$
$$\log Y_1^* = (1-\alpha)\left[\eta \log N_1^* + (1-\eta)\log N_1\right]$$

Therefore, it is easy to show that the asset prices as well as the real activities in the two countries are not perfectly correlated.

6 The OLG Model

This section extends our baseline model to the OLG model. It serves two purposes. First, it shows that our sentiment-driven equilibria are robust to a dynamic setting. Second, it illustrates that the sentiment-driven fluctuations hold the promise to explain actual business cycle fluctuations, as the sentiment shocks can generate persistent fluctuations in output and employment, a defining feature of all observed business cycles.

Timeline In each period t, there are four stages:

Stage 1: The old generation of workers become capitalists (entrepreneurs) and a new generation of workers is born. Capitalists and workers are informed of the history of $A^t = \{A_\tau\}_{\tau=0}^t$. Only capitalists receive private signals about A_{t+1} to be realized in the next period.

Stage 2: Capitalists trade capital among themselves in a financial market before production based on their private signals on A_{t+1}, the history information A^t, the sentiment shock z_t, and the capital price P_t.

Stage 3: Based on their capital stock, wage w_t and productivity A_t, capitalists hire workers and produce. Workers decide their labour supply. Workers obtain information about A_{t+1} through prices P_t and w_t.

Stage 4: Capitalists consume and then die. Workers save their labour income for the next period as their capital. The economy repeats stage 1 to 4 in the next period.

Capitalists (entrepreneurs) Let us first consider the problem of capitalist j who receives a private signal $s_{jt} = \log A_{t+1} + \varepsilon_{jt}$. He solves

$$V_t(\varepsilon_{jt}, K_t) = \max_{K_{jt}} \mathbb{E}\left[C_{jt}|P_t, A^t, s_{jt}, z_t\right] \tag{49}$$

where

$$C_{jt} = P_t M_{jt} + \max_{N_{jt}} \left[A_t(K_{jt} - M_{jt})^\alpha N_{jt}^{1-\alpha} - w_t N_{jt}\right].$$

In stage 2, the capitalist can sell $M_{jt} \leq K_t$ to other capitalists and keeps $K_t - M_{jt}$ for production. In stage 3, the capitalist chooses to hire labour, N_{jt}, and to produce with production function $Y_{jt} = A_t(K_{jt} - M_{jt})^\alpha N_{jt}^{1-\alpha}$.

We work by backward induction, from stage 3 to stage 2. In stage 3, given K_t, M_{jt}, w_t and A_t, capitalist j's first order condition with respect to N_{jt} is

$$N_{jt} = \left[\frac{A_t(1-\alpha)}{w_t}\right]^{\frac{1}{\alpha}} (K_{jt} - M_{jt}) \tag{50}$$

and hence the revenue net of labour cost is

$$C_{jt} = P_t M_{jt} + \alpha A_t \left[\frac{A_t(1-\alpha)}{w_t}\right]^{\frac{1-\alpha}{\alpha}} (K_{jt} - M_{jt}). \tag{51}$$

We then move to stage 2. Let $R_t \equiv \alpha A_t \left[\frac{A_t(1-\alpha)}{w_t}\right]^{\frac{1-\alpha}{\alpha}}$. It follows that

$$V_t(\varepsilon_{jt}, K_{jt}) = \mathbb{E}\left[C_{jt}|P_t, A^t, s_{jt}, z_t\right] = \mathbb{E}\left[P_t M_{jt} + R_t(K_{jt} - M_{jt})|P_t, A_t, a_{t+1} + \varepsilon_{jt}, z_t\right]$$

Capitalist j's first order condition with respect to M_{jt} is

$$P_t = \mathbb{E}\left[R_t|P_t, A_t, a_{t+1} + \varepsilon_{jt}, z_t\right]. \tag{52}$$

It follows that

$$V_t(\varepsilon_{jt}, K_{jt}) = P_t K_{jt}. \tag{53}$$

Workers Workers are assumed to be homogenous with utility function $C_{t+1} - \psi \frac{N_t^{1+\gamma}}{1+\gamma}$, where C_{t+1} is a worker's consumption in period $t+1$ and N_t is his labor supply in period t . Let K_{t+1} be his saving at the end of period t. The worker will become an entrepreneur in the next period with private information on $s'_{t+1} = a_{t+2} + \varepsilon'$. His expected consumption perceived at the beginning of the next period is thus given by $V_{t+1}(\varepsilon', K_{t+1})$ defined by equation (49). Equation (53) implies $V_{t+1}(\varepsilon', K_{t+1}) = P_{t+1}K_{t+1}$. The worker's problem thus can be written as

$$\mathbb{E}[P_{t+1}K_{t+1} - \psi \frac{N_t^{1+\gamma}}{1+\gamma}|w_t, P_t, A_t] \tag{54}$$

with budget constraint

$$K_{t+1} = w_t N_t.$$

The first order condition of (54) with respect to N_t is

$$\begin{aligned} \psi N_t^{\gamma} &= w_t \mathbb{E}\left[P_{t+1}|w_t, P_t, A_t\right] \\ &= w_t \mathbb{E}\left\{\mathbb{E}\left[R_{t+1}|P_{t+1}, A_{t+1}, a_{t+2} + \varepsilon'\right]|P_t, A_t, K_t\right\} \\ &= w_t \mathbb{E}\left[R_{t+1}|P_t, A_t, K_t\right] \end{aligned} \tag{55}$$

The last equality in (55) follows by the law of iterated expectations. Also, in (55), since w_t is a function of N_t, the information set $\{w_t, P_t, A_t\}$ is effectively equivalent to $\{P_t, A_t\}$.

In equilibrium, we have

$$\int M_{jt} dj = 0.$$

As all the entrepreneurs start with the same level of capital, $K_{jt} = w_{t-1}N_{t-1} = K_t$. By (50), the labor market equilibrium condition thus can be written as

$$N_t = \int N_{jt} dj = \left[\frac{A_t(1-\alpha)}{w_t}\right]^{\frac{1}{\alpha}} K_t. \tag{56}$$

From the result in (51), the total production can be written as

$$Y_t = \int A_t \left[\frac{A_t(1-\alpha)}{w_t}\right]^{\frac{1-\alpha}{\alpha}} (K_{jt} - M_{jt}) dj = \left[\frac{A_t(1-\alpha)}{w_t}\right]^{\frac{1-\alpha}{\alpha}} A_t K_t. \tag{57}$$

Equations (56) and (57) together imply

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha},$$

$$w_t = (1-\alpha) A_t K_t^{\alpha} N_t^{-\alpha} \text{ and } R_t = \alpha \frac{Y_t}{K_t}. \tag{58}$$

Capital evolves as

$$K_{t+1} = (1-\alpha) Y_t = (1-\alpha) A_t K_t^{\alpha} N_t^{1-\alpha}. \tag{59}$$

Equation (55) becomes

$$\psi N_t^{\gamma+1} = \alpha \mathbb{E}\left[Y_{t+1} | P_t, A_t\right] = \alpha \mathbb{E}\left\{A_{t+1} N_{t+1}^{1-\alpha} \left[(1-\alpha) A_t K_t^{\alpha} N_t^{1-\alpha}\right]^{\alpha} | P_t, A_t\right\} \tag{60}$$

Finally, the capital price must be equal to the fundamental value, that is,

$$P_t = \mathbb{E}\left[R_t | P_t, A_t, a_{t+1} + \varepsilon_{jt}\right] = R_t, \tag{61}$$

as in the benchmark model.

By normalizing $\psi^{-1}\alpha(1-\alpha)^{\alpha} = 1$ and recalling $\theta = \frac{1}{\gamma+1-(1-\alpha)\alpha}$ defined in (10), we obtain a key set of equations in equilibrium (we denote $x_t = \log X_t$).

$$\begin{aligned}
n_t &= \alpha\theta a_t + \alpha^2 \theta k_t + \theta \log \mathbb{E}\left[\exp\left(a_{t+1} + (1-\alpha) n_{t+1}\right) | p_t, a_t, k_t\right] && (62)\\
k_{t+1} &= \log(1-\alpha) + a_t + \alpha k_t + (1-\alpha) n_t && (63)\\
p_t &= \mathbb{E}\left[\log \alpha + a_t + (1-\alpha)(n_t - k_t) | p_t, a_t, k_t, a_{t+1} + \varepsilon_{jt}, z_t\right] && (64)
\end{aligned}$$

where (62) is from (60), (63) is from (59), and (64) is from (61). Here we have used the fact that the relevant economy history up to period t can be summarized by a_t and k_t.

We conjecture that equilibrium labor takes the form

$$n_t = n^c + \varphi a_t + \pi k_t + (\phi a_{t+1} + \sigma_z z_t),$$

where n^c, φ, π, ϕ, and σ_z are undetermined coefficients. Proposition 8 summarizes the equilibria.

Proposition 8 *There exists a continuum of equilibria indexed by $0 \leq \sigma_z^2 \leq \frac{\hat{\theta}^2}{4}\sigma_a^2$, in which the price P_t is given by*

$$p_t = [\log \alpha + (1-\alpha) n^c] + [1 + (1-\alpha)\varphi] a_t + (1-\alpha)(\pi - 1) k_t + (1-\alpha)(\phi a_{t+1} + \sigma_z z_t),$$

and

$$n_t = n^c + \varphi a_t + \pi k_t + (\phi a_{t+1} + \sigma_z z_t)$$

$$k_{t+1} = \log(1-\alpha) + a_t + \alpha k_t + (1-\alpha) n_t$$

$$y_t = a_t + \alpha k_t + (1-\alpha) n_t$$

where

$$\pi = \frac{\left[\frac{1}{(1-\alpha)^2\theta} - \frac{\alpha}{1-\alpha}\right] - \sqrt{\left[\frac{1}{(1-\alpha)^2\theta} - \frac{\alpha}{1-\alpha}\right]^2 - 4\left(\frac{\alpha}{1-\alpha}\right)^2}}{2},$$

$$\varphi = \frac{\alpha\theta + (1-\alpha)\pi\theta}{1 - (1-\alpha)^2\pi\theta},$$

$$\phi = \frac{\hat{\theta} \pm \sqrt{\hat{\theta}^2 - \frac{4\sigma_z^2}{\sigma_a^2}}}{2},$$

and

$$n^c = \frac{\theta(1-\alpha)\pi\log(1-\alpha) + \frac{1}{2}\theta\sigma_a^2 \begin{bmatrix} \begin{pmatrix} [(1-\alpha)\phi]^2 \\ -[(1-\alpha)\phi] \end{pmatrix} \\ +\left(1 - \frac{\phi}{\hat{\theta}}\right)\begin{pmatrix} (1+(1-\alpha)\varphi)^2 \\ -(1+(1-\alpha)\varphi) \end{pmatrix} \end{bmatrix} + \frac{1}{2}\theta \begin{pmatrix} [(1-\alpha)\sigma_z]^2 \\ -[(1-\alpha)\sigma_z] \end{pmatrix}}{1 - \theta(1-\alpha)[1 + \pi(1-\alpha)]}.$$

and we define $\hat{\theta} = \frac{\theta[1+(1-\alpha)\varphi]}{1-\theta(1-\alpha)^2\pi}$.

Proof. *See appendix.* ■

The intuition is similar to that of our benchmark model. While a_{t+1} directly affects the workers' return on savings, its effect on existing entrepreneurs (old generation) is only through an indirect general equilibrium channel. If high sentiments lead entrepreneurs to speculate that the output and hence the demand for capital will be high in period t, competition will drive up the capital price P_t. After observing a high price P_t, the workers in period t need to solve a signal extraction problem, leading them to attribute the high price P_t partially to high productivity in the next period. High productivity in the next period increases their incentive to supply labor in the current period. So indeed output and the demand for capital will be high in period t. The initial conjecture of the existing entrepreneurs is verified.

7 Conclusion

In this paper, we study how the financial sector can affect the aggregate real economy through the information channel. In the rational expectation framework, we show that investors' sentiments affect financial market prices which in turn influence real activities. Because of the two-way feedback between the financial sector and the real sector, a small sentiment shock in the financial market can be amplified, and can have a large impact on the real economy. The sentiment-driven equilibria can also result in cross-country comovements in asset prices and real output. Under informational frictions, investors' perceived synchronization across economies can lead to the actual synchronization. In a dynamic economy, sentiment-driven fluctuations can also generate persistence in business cycles.

Appendix

A Proofs

Proof of Proposition 3: The first condition, (10), becomes

$$\begin{aligned}\log N_1 &= \theta \log \{\mathbb{E}[A_2|\phi a_2 + \sigma_z z]\} \\ &= \theta\left\{\mathbb{E}[a_2|\phi a_2 + \sigma_z z] + \frac{1}{2} var(a_2|\phi a_2 + \sigma_z z)\right\}\end{aligned}$$

We have

$$\mathbb{E}[a_2|\phi a_2 + \sigma_z z] = -\frac{1}{2}\sigma_a^2 + \frac{\phi\sigma_a^2}{\phi^2\sigma_a^2 + \sigma_z^2}\left(\phi a_2 + \sigma_z z + \frac{1}{2}\phi\sigma_a^2\right),$$

and

$$var(a_2|\phi a_2 + \sigma_z z) = \sigma_a^2 - \frac{\phi^2\sigma_a^4}{\phi^2\sigma_a^2 + \sigma_z^2}.$$

Comparing coefficients with the conjecture $\log N_1 = \bar{n} + \phi a_2 + \sigma_z z$ yields

$$\phi a_2 + \sigma_z z = \frac{\theta\phi\sigma_a^2}{\phi^2\sigma_a^2 + \sigma_z^2}\left(\phi a_2 + \sigma_z z\right),$$

or

$$1 = \frac{\theta\phi\sigma_a^2}{\phi^2\sigma_a^2 + \sigma_z^2}, \tag{A.1}$$

and

$$\bar{n} = \theta\left\{-\frac{1}{2}\sigma_a^2 + \frac{\phi\sigma_a^2}{\phi^2\sigma_a^2 + \sigma_z^2}\frac{1}{2}\phi\sigma_a^2 + \frac{1}{2}\sigma_a^2 - \frac{1}{2}\frac{\phi^2\sigma_a^4}{\phi^2\sigma_a^2 + \sigma_z^2}\right\} = 0.$$

Solving (A.1) with respect to ϕ gives

$$\phi = \frac{\theta}{2} \pm \frac{\sqrt{\theta^2\sigma_\alpha^2 - 4\sigma_z^2}}{2\sigma_\alpha}$$

where $0 \leq \sigma_z^2 \leq \frac{\theta^2}{4}\sigma_a^2$.

Equation (11) becomes

$$\log P_0 = \log\alpha + (1-\alpha)\log N_1 = \log\alpha + (1-\alpha)\left(\phi a_2 + \sigma_z z\right),$$

that is, $\bar{p} = 0$.

Proof of Proposition 4: We show that the combination of (20), (23) and (22) satisfies conditions (17) and (18). Note that W_1 is a function of N_1, so given (22) we must have (21). First, because W_1 is in the form of (21), a household can infer a_2^H perfectly by comparing W_1 with P_0. By the argument in section 4.1.1, the effect of the information set of the households becomes the same across households, namely $\Omega_{i1} = \Omega_1 = \{R_1, W_1, P_0, a_2^H\}$. So households can make an identical decision in period 1; that is, symmetric equilibrium among households, condition (17), still applies.

Condition (17) becomes

$$\begin{aligned}
\log N_1 &= \theta \log \left\{ \mathbb{E}\left[A_2 | \phi a_2^I + \sigma_z z, a_2^H\right] \right\}, \\
&= \theta \left\{ \mathbb{E}\left[a_2 | \phi a_2^I + \sigma_z z, a_2^H\right] + \frac{1}{2} var(a_2 | \phi a_2^I + \sigma_z z, a_2^H) \right\} \\
&= \theta \left\{ \mathbb{E}\left[a_2^I + a_2^H | \phi a_2^I + \sigma_z z, a_2^H\right] + \frac{1}{2} var(a_2^I + a_2^H | \phi a_2^I + \sigma_z z, a_2^H) \right\} \\
&= \theta \left\{ \mathbb{E}\left[a_2^I | \phi a_2^I + \sigma_z z\right] + \frac{1}{2} var(a_2^I | \phi a_2^I + \sigma_z z) \right\} + \theta a_2^H \qquad \text{(A.2)}
\end{aligned}$$

We have

$$\mathbb{E}\left[a_2^I | \phi a_2^I + \sigma_z z\right] = -\frac{1}{2}\sigma_I^2 + \frac{\phi \sigma_I^2}{\phi^2 \sigma_I^2 + \sigma_z^2} \left(\phi a_2^I + \sigma_z z + \frac{1}{2} \phi \sigma_I^2 \right)$$

and

$$var(a_2^I | \phi a_2^I + \sigma_z z) = \sigma_I^2 - \frac{\left(\phi \sigma_I^2\right)^2}{\phi^2 \sigma_I^2 + \sigma_z^2}$$

Under condition (23), it is easy to verify that (A.2) becomes (22).

Next, we turn to the equilibrium condition of (18), which becomes

$$\begin{aligned}
\log P_0 &= \log \alpha + \log \mathbb{E}\left[\exp\left[(1-\alpha)\log N_1\right] | \phi a_2^I + \sigma_z z, a_2^I + \varepsilon_j\right] \\
&= \log \alpha + \mathbb{E}\left[(1-\alpha)\log N_1 | \phi a_2^I + \sigma_z z, a_2^I + \varepsilon_j\right] + \frac{1}{2} var\left((1-\alpha)\log N_1 | \phi a_2^I + \sigma_z z, a_2^I + \varepsilon_j\right) \\
&= \log \alpha + \mathbb{E}\left[(1-\alpha) \begin{pmatrix} (\phi a_2^I + \sigma_z z) \\ +\theta a_2^H \end{pmatrix} | \phi a_2^I + \sigma_z z, a_2^I + \varepsilon_j \right] \\
&\quad + \frac{1}{2} var\left((1-\alpha) \begin{pmatrix} (\phi a_2^I + \sigma_z z) \\ +\theta a_2^H \end{pmatrix} | \phi a_2^I + \sigma_z z, a_2^I + \varepsilon_j \right) \\
&= \log \alpha + (1-\alpha)\left(\phi a_2^I + \sigma_z z\right)
\end{aligned}$$

This condition is true by (20).

Proof of Proposition 5: We first simplify equation (36) to

$$\psi N_1^\gamma = \frac{(1-\alpha)Y_1}{N_1} \mathbb{E}\left[\frac{\alpha Y_2}{(1-\alpha)Y_1} | P_0, W_1, R_1, P_0^*\right] = \frac{\alpha}{N_1} \mathbb{E}[Y_2 | P_0, W_1, R_1, P_0^*]$$

Then we substitute Y_2 by (34) and $K_2 = (1-\alpha)Y_1 = (1-\alpha)N_1^{1-\alpha}$ to obtain

$$\begin{aligned} N_1^{\gamma+1} &= \mathbb{E}[A_2^\eta Y_1^{\alpha\eta} A_2^{*1-\eta} Y_1^{*\alpha(1-\eta)}|\Omega_1] \\ &= \left[\left(N_1^{1-\alpha}\right)^\eta (N_1^{*1-\alpha})^{1-\eta}\right]^{\alpha\eta} \left[\left(N_1^{*1-\alpha}\right)^\eta (N_1^{1-\alpha})^{1-\eta}\right]^{\alpha(1-\eta)} \mathbb{E}[A_2^\eta A_2^{*1-\eta}|\Omega_1]. \end{aligned}$$

Re-arranging terms yields equation (40), and equation (41) follows from the symmetry.

Considering

$$\begin{aligned} R_1 &= \alpha Y_1 = \alpha\left(N_1^{1-\alpha}\right)^\eta \left(N_1^{*1-\alpha}\right)^{1-\eta} \\ R_1^* &= \alpha Y_1^* = \alpha\left(N_1^{*1-\alpha}\right)^\eta \left(N_1^{1-\alpha}\right)^{1-\eta}, \end{aligned}$$

we also obtain (42) and (43).

Proof of Proposition 7: First, given (44), the conjecture in equation (40) yields

$$\log N_1 = \frac{\lambda}{1-\omega}\log\mathbb{E}\left[A_2^\eta A_2^{*1-\eta}|(\phi(a_2+a_2^*)+\sigma_z(z+z^*)\right]$$

Notice that

$$\begin{aligned} & \log\mathbb{E}\left[A_2^\eta A_2^{*1-\eta}|(\phi(a_2+a_2^*)+\sigma_z(z+z^*)\right] \\ = {} & \mathbb{E}\left[\eta a_2+(1-\eta)a_2^*|(\phi(a_2+a_2^*)+\sigma_z(z+z^*)\right] \\ & +\frac{1}{2}var\left[\eta a_2+(1-\eta)a_2^*|(\phi(a_2+a_2^*)+\sigma_z(z+z^*)\right] \end{aligned}$$

where

$$\begin{aligned} & \mathbb{E}\left[\eta a_2+(1-\eta)a_2^*|(\phi(a_2+a_2^*)+\sigma_z(z+z^*)\right] \\ = {} & -\frac{1}{2}\sigma_a^2+\frac{\phi\sigma_a^2}{2\phi^2\sigma_a^2+2\sigma_z^2}\left[\phi(a_2+a_2^*)+\sigma_z(z+z^*)+\phi\sigma_a^2\right] \end{aligned}$$

and

$$\begin{aligned} & var\left[\eta a_2+(1-\eta)a_2^*|(\phi(a_2+a_2^*)+\sigma_z(z+z^*)\right] \\ = {} & \left[\eta^2+(1-\eta)^2\right]\sigma_a^2-\frac{\left(\phi\sigma_a^2\right)^2}{2\phi^2\sigma_a^2+2\sigma_z^2} \end{aligned}$$

Thus, by comparing coefficients yields, we obtain

$$\phi(a_2+a_2^*)+\sigma_z(z+z^*) = \frac{\lambda}{1-\omega}\frac{\phi\sigma_a^2}{2\phi^2\sigma_a^2+2\sigma_z^2}\left[\phi(a_2+a_2^*)+\sigma_z(z+z^*)\right]$$

or

$$\frac{\lambda}{1-\omega}\frac{\phi\sigma_a^2}{2\phi^2\sigma_a^2+2\sigma_z^2}=1.$$

and

$$\overline{\overline{n}}=\frac{\lambda}{2\left(1-\omega\right)}\left\{\left[\eta^2+\left(1-\eta\right)^2-1\right]+\frac{\phi^2\sigma_a^2}{2\phi^2\sigma_a^2+2\sigma_z^2}\right\}\sigma_a^2$$

Second, given (45), it is easy to confirm that financial prices are (44).

By symmetry, we can also verify the equilibrium for the foreign country.

Proof of Proposition 8: Given the price p_t, the workers' effective information set is $\{a_t, k_t, \phi a_{t+1}+\sigma_z z_t\}$. Equation (62) becomes

$$n_t=\alpha\theta a_t+\alpha^2\theta k_t+\theta\log\mathbb{E}\left[\exp\left(a_{t+1}+\left(1-\alpha\right)n_{t+1}\right)|k_t,a_t,\phi a_{t+1}+\sigma_z z_t\right]$$

or

$$n_t=\alpha\theta a_t+\alpha^2\theta k_t+\frac{1}{2}\theta\Phi_t+\theta\mathbb{E}\left[a_{t+1}+\left(1-\alpha\right)n_{t+1}|k_t,a_t,\phi a_{t+1}+\sigma_z z_t\right],$$

where $\Phi=var(a_{t+1}+\left(1-\alpha\right)n_{t+1}|k_t,a_t,\phi a_{t+1}+\sigma_z z_t)$. We can calculate

$$\begin{aligned} n_t &= \theta(1-\alpha)\left[n^c+\pi\log(1-\alpha)\right]+\frac{1}{2}\theta\Phi \\ &\quad+\left[\alpha\theta+\theta(1-\alpha)\pi\right]a_t+\left[\alpha^2\theta+\theta(1-\alpha)\pi\alpha\right]k_t \\ &\quad+\theta(1-\alpha)^2\pi n_t \\ &\quad+\theta\left[-\frac{1}{2}\left[1+(1-\alpha)\varphi\right]\sigma_a^2+\frac{\left[1+(1-\alpha)\varphi\right]\phi\sigma_a^2}{\phi^2\sigma_a^2+\sigma_z^2}\left(\phi a_{t+1}+\sigma_z z_t+\frac{1}{2}\phi\sigma_a^2\right)\right]. \end{aligned}$$

Comparing terms regarding n_t yields

$$\pi=\frac{\alpha^2\theta+\theta(1-\alpha)\alpha\pi}{1-\theta(1-\alpha)^2\pi}$$

or

$$\pi=\frac{\left[\frac{1}{(1-\alpha)^2\theta}-\frac{\alpha}{1-\alpha}\right]-\sqrt{\left[\frac{1}{(1-\alpha)^2\theta}-\frac{\alpha}{1-\alpha}\right]^2-4\left(\frac{\alpha}{1-\alpha}\right)^2}}{2}.$$

And φ is given by

$$\varphi=\frac{\alpha\theta+(1-\alpha)\pi\theta}{1-(1-\alpha)^2\pi\theta}.$$

As for ϕ, we have

$$1=\theta\frac{\left[1+(1-\alpha)\varphi\right]}{1-\theta(1-\alpha)^2\pi}\frac{\phi\sigma_a^2}{\phi^2\sigma_a^2+\sigma_z^2}.$$

We define $\hat{\theta} = \frac{\theta[1+(1-\alpha)\varphi]}{1-\theta(1-\alpha)^2\pi}$. Thus,

$$
\begin{aligned}
1 &= \hat{\theta}\frac{\phi\sigma_a^2}{\phi^2\sigma_a^2+\sigma_z^2} \\
\implies \quad \phi &= \frac{\hat{\theta} \pm \sqrt{\hat{\theta}^2 - \frac{4\sigma_z^2}{\sigma_a^2}}}{2}.
\end{aligned}
$$

Once we have ϕ, we can solve the constant n^c.

$$
n^c = \frac{\theta(1-\alpha)\pi\log(1-\alpha) + \frac{1}{2}\theta\sigma_a^2 \left[\begin{array}{c} \left(\begin{array}{c} [(1-\alpha)\phi]^2 \\ -[(1-\alpha)\phi] \end{array} \right) \\ +\left(1-\frac{\phi}{\hat{\theta}}\right)\left(\begin{array}{c} (1+(1-\alpha)\varphi)^2 \\ -(1+(1-\alpha)\varphi) \end{array} \right) \end{array} \right] + \frac{1}{2}\theta \left(\begin{array}{c} [(1-\alpha)\sigma_z]^2 \\ -[(1-\alpha)\sigma_z] \end{array} \right)}{1-\theta(1-\alpha)\left[1+\pi(1-\alpha)\right]}.
$$

If we set $\sigma_z = 0$, then ϕ has two solutions, 0 and $\hat{\theta}$. It is easy to verify that the sentiment-driven equilibrium in Proposition 8 is identical to the perfectly revealing equilibrium that corresponds to $\sigma_z = 0$ and $\phi = \hat{\theta}$ and is identical to the non-revealing equilibrium that corresponds to $\sigma_z = 0$ and $\phi = 0$.

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