On Ratifiability of Efficient Cartel Mechanisms in First-Price Auctions with Participation Costs and Information Leakage

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Introduction

- Auction is an effective way to extract private information by improving the competitiveness of potential buyers and thus can increase allocation efficiency from the perspectives of both sellers and the social planner.
- Efficiency, however, is diminished when buyers' collusion occurs.

Introduction

A cartel faces one external and four internal problems.

- A cartel has to anticipate and prevent outside production in order to avoid external threat.
- The four internal problems are designing the rule, dividing the profit, detecting, and deterring cheating.

McAfee and McMillan (1992) establish an incentive compatible and efficient cartel mechanism, which is ratifiable in the first-price sealed-bid auction format.

In this paper, I investigate the ratifiable and nonratifiable conditions of a cartel mechanism with auction format.

Cartel Mechanism

The timing of possible cartel formation between date 0 and date 1:

- At date 0, a cartel's mechanism exists and all bidders belong to the cartel.
- At date $\frac{1}{4}$, nature draws a private valuation for each bidder.
- At date $\frac{2}{4}$, each bidder reports his bid in cartel's prior auction.
- At date $\frac{3}{4}$, bidders update their beliefs through the cartel's auction and vote for or against the collusive mechanism.
- At date 1, if all bidders accepted the collusive mechanism at date $\frac{3}{4}$, the winner in the prior auction represents the cartel to bid in the seller's auction, and he will compensate the losers with transfer payments. If at least one bidder rejects the collusive mechanism, no collusion occurs. Bidders bid in the seller's auction at date 1 noncooperatively.

Example

Table: Side payments from a knockout cartel

Knockout auction	Bid	Side payments
Bidder A	8,000	$\frac{8,000-5,000}{4} = 750$
Bidder B	7,000	$\frac{8,000-5,000}{4} = 750$
Bidder C	6,000	$\frac{8,000-5,000}{4} = 750$
Bidder D	3,000	$\frac{8,000-5,000}{4} = 750$
Target auction price	5,000	

This cartel is ratifiable because everyone in the cartel is better off than they would be in the non-collusive case.

Literatures

Model	Cartel	Auction	Cost	Supported
McAfee; McMillan (1992)	Yes	First Price	No	Yes
Tan; Yilankaya (2007)	Yes	Second Price	Yes	No
Cao; Tian (2010)	No	First Price	Yes	—
This Paper	Yes	First Price	Yes	No

Difference

Unlike McAfee and McMillan (1992) studying the coordinated bidding strategies in a strong cartel, I investigate this efficient collusive mechanism with two important conditions:

- Bidders can update their information through a cartel's prior auction.
- They have to pay participation costs to participate in seller's auction.

I discover that the efficient cartel mechanism, such as pre-auction knockout, could be supported when either the two conditions exists. However, this mechanism is no longer sustainable when both conditions exist.

Economic Environment

- r: the only seller's reserve price, $r \in [0, 1]$.
- v_i: bidder i's valuation, $(i = 1, ..., n)$, $v_i \in [0, 1]$.
- c: participation costs, $c \in [0, 1)$.
- $\pi(\cdot)$: the profit function.
- $F(\cdot)$: the distribution function, $F(v_i)^{n-1} = G(v_i)$.

Bidders do not know others' participation decisions when they make their own decisions.

Non-Collusive Auction

The bidding function $\gamma(v_i)$ in first-price sealed-bid auction can be written as

$$\gamma(v_i) = \frac{\int_{v^*}^{v_i} y d[F(y)]^{n-1}}{[F(v_i)]^{n-1}},$$

where v^* is the cutoff point, which is determined by $c = v^* F(v^*)^{n-1}$, $v^* > c$.

The noncollusive profit $\pi_i^s(v_i)$ for bidder i is as follows:

$$\pi_i^s(v_i) = \begin{cases} 0 & v_i < v^* \\ \int_{v^*}^{v_i} [F(y)]^{n-1} dy & v_i \geq v^*. \end{cases}$$

Reminder

- The seller's behavior is passive in this paper.
- The cartel mechanism is designed to maximize the ex ante sum of bidders' expected profits in the auction, which means this cartel is self-enforcing.
- Two auctions. One is cartel's prior auction, the other is seller's legit auction. If the cartel is formed, the bidder with the highest bid in the cartel's auction would be chosen to bid in the seller's auction.

Efficient Cartel Mechanism

Bidder i's payoff function is $\pi_i^m(v_i)$, and $\pi_i^m(0)$ is the transfer payment received by each cartel member.

$$\pi_i^m(v_i) = \begin{cases} \pi_i^m(0) & v_i < c. \\ \pi_i^m(0) + \int_c^{v_i} G(y)dy & v_i \geq c, \end{cases} \tag{1}$$

where

$$\pi_i^m(0) = \int_c^1 [y - \frac{1 - F(y)}{f(y)} - c]G(y)dF(y).$$

Since $v^* \geq c$, and $\pi_i^m(0) \geq 0$, I obtain

$$\pi_i^m(v_i) \geq \pi_i^s(v_i) \quad \forall v_i \in [0, 1].$$

Example

Table: Side payments from an efficient cartel with \$0 reserve price

Knockout auction	Bid	Side payments
Bidder A	8,000	$\frac{8,000-0}{4} = 2,000$
Bidder B	7,000	$\frac{8,000-0}{4} = 2,000$
Bidder C	6,000	$\frac{8,000-0}{4} = 2,000$
Bidder D	3,000	$\frac{8,000-0}{4} = 2,000$
Reserve Price	0	
Participation costs	1,000	

Definition

- Cramton and Palfrey (1995) characterize the information leakage problem from participation decisions in standard mechanism design approach.
- If bidders are making interim participation decisions for the collusive mechanism, others can make inferences about a bidder's type from his choice between the collusive mechanism and the seller's auction, which in turn may affect bidders' payoff.

Definition

The information leakage problem is that all bidders can update their information through the cartel's prior auction before they participate in the seller's auction.

Proposition

Proposition

Suppose there is no the information leakage problem. Then the strong cartel mechanism is efficient and interim individually rational with respect to symmetric equilibrium payoffs in the seller's auction no matter whether there is a participation cost.

Definition

- Assumption: when a bidder is indifferent between staying in and vetoing for the cartel, he would choose to stay in the cartel.
- A veto set A. If the vetoer's value is in this set, he will veto for the cartel, that is, vetoing for the cartel brings higher profit to the vetoer than in collusive case.

Definition

A set A_i with $\emptyset \neq A_i \subseteq [0,1]$ for bidder i is said to be a credible veto set if there exists an equilibrium b in the post-veto auction such that $\pi_i^v(v_i, b) > \pi_i^m(v_i) \Leftrightarrow v_i \in A_i$.

Definition

Suppose when one of the bidders vetoes the cartel, others believe that his value is in $(v_N, 1]$, so $A = (v_N, 1]$ is a credible veto set for any bidder i.

Definition

The cartel mechanism is ratifiable, if there is no credible veto set for all $i \in N$.

Following Cao and Tian (2010), the bidder i's bid b can be determined by the following maximization problem:

$$\max_b F(v_j(b))^{n-1}(v_i - b).$$

Similarly, a bidder j's bid can be solved by the following problem:

$$\max_b F(v_i(b))F(v_j(b))^{n-2}(v_j - b).$$

The optimal inverse bidding functions, $v_i(b)$ and $v_j(b)$ when participating, are uniquely given by the first order conditions: For all $\underline{b} < b \leq \overline{b}$,

$$v_i(b) = b + \frac{F(v_j(b))}{(n-1)f(v_j(b))v_j'(b)}, \tag{2}$$

$$v_j(b) = b + \frac{F(v_i(b))F(v_j(b))}{(n-2)f(v_j(b))v_j'(b)F(v_i(b)) + F(v_j(b))f(v_i(b))v_i'(b)}, \tag{3}$$

with boundary conditions $v_i(\underline{b}) = v_N$, $v_j(\underline{b}) = \underline{b}$, $v_i(\overline{b}) = v_j(\overline{b}) = \overline{b} = 1$.

Remark

Remark

The information structure we adopted is as Menezes and Monteiro (2000). A bidder does not know who else is in the auction when he submits a bid which is a different specification with Cao and Tian (2010). But in Menezes and Monteiro (2000), they only focus on the symmetric equilibrium at which all bidders use the same cutoff point (which is equal to v^*) and submit bids via the same bidding function. We focus on the asymmetric equilibria when bidders use different cutoff points.

Ratifiability

Define $Q(v_i(b)) \equiv v_j(v_i)$, where $Q(v_i(b))$ is the relationship between v_j and v_i when $b = b_j = b_i$, and $k(v_i)$ is the probability that given $v_i > v_N$, the vetoer i's bid is greater than bidder j's.

$$v_j(b) = b + \frac{F(v_i(b))}{(\frac{n-2}{n-1})(\frac{F(v_i(b))}{v_i(b)-b}) + f(v_i(b))v_i'(b)}. \tag{4}$$

$$\begin{aligned} k(v_i) &= P(b_i > b_j | v_i > v_N) \\ &= \frac{\int_{v_N}^{1} f(v_i) \int_{0}^{Q(v_i)} f(v_j) dv_j dv_i}{1 - F(v_N)} \\ &= F(Q(v_i)). \end{aligned}$$

Ratifiability

- Let $H(v_i) = k(v_i)^{n-1}$ be the probability that all other bidders' bids are less than vetoer i's.

- For any given bidder in the cartel, the maximum of others' bids is given by the distribution $\hat{H}(y) \equiv k(y)F(y)^{n-2}$, with $y \in (v_N, 1]$.

Let $\tilde{v}_Y$ be the solution to

$$[\tilde{v}_Y - b^*_{-i}(\tilde{v}_Y)]\hat{H}(\tilde{v}_Y) = c.$$

Ratifiability

- The payoff of a $\tilde{v}_Y$ type bidder is equal to his participation costs, whenever $\tilde{v}_Y \leq 1$. We have $v_Y = min\{1, \tilde{v}_Y\}$.

- v_Y is the cutoff point where other bidders are indifferent between staying and vetoing the cartel. An increase in v_N leads to a higher v_Y.

The payoff of vetoer i is

$$\pi_i^v(v_i, b^*) = max\{[v_i - \lambda_i(v_i)]H(v_i) - c, 0\}.$$

Ratifiability

Consider the simple case where $c = 0$ with information leakage problem. With the information gained through the cartel's mechanism, if there is a vetoer of the cartel after the cartel's auction, other bidders can enter the seller's auction for free and bid as much as possible to make the vetoer earn a profit that is less than or equal to his cartel revenue, which means:

$$\pi_i^m(v_i) \geq \pi_i^v(v_i, b^*) = \pi_i^s(v_i).$$

Proposition

The efficient cartel mechanism is ratifiable when $c = 0$, even if the information leakage problem exists.

Remark

Remark

This result is opposite to the one claimed in Tan and Yilankaya (2007). On page 390, they indicated that: "*Notice that* $v_N^* > 0$. *This implies that even when* $c = 0$ *the efficient cartel mechanism is not ratifiable.*" However, they made the claim without proof. Actually, they are even not quite sure about their claim without further investigating why. In contrast, we show that when $c = 0$, the cartel mechanism is ratifiable. This is because the vetoer's betraying signal becomes an "incredible threat" without participation costs.

Ratifiability

Consider the general case where participation costs are positive and the information leakage problem exists.

Having updated their beliefs that the vetoer's value belongs in A, other bidders do not participate in seller's auction since they have to pay the non-refundable participation costs and earn a negative profit. The cartel mechanism is not ratifiable.

Proposition

In first-price sealed-bid auction, suppose $c > 0$ and the information leakage problem exists. Then the strong efficient cartel mechanism is no longer ratifiable.

Example

Table: Bidders payoff for a broken cartel

Knockout auction	Bid	Side payments
Bidder A	8,000	$8,000 - 1,000 = 7,000$
Bidder B	7,000	0
Bidder C	6,000	0
Bidder D	3,000	0
Target auction price	0	
Participation costs	1,000	

Remark

Remark

When bidders have different participation costs, which means that bidders report the level at their values minus the participation costs, the highest-value bidder may not be the winner in a cartel because he may have higher participation costs. As in the Proposition, we can get a similar result: when the information leakage problem exists, the bidder with the largest net value which is defined by the difference between his value and his participation cost would have an incentive to betray the cartel to maximize his revenue.

Future research

We would like to introduce bribery to the model and examine what third-party authority's strategies might destabilize the cartel. If the cartel can bribe the government (politician), is the cartel mechanism efficient and ratifiable? It may require more social costs to differentiate the collusive behavior.

THANK YOU!