Risk Aversion in the Nash Bargaining Problem with Uncertainty

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Abstract

We apply the aggregation property of Identical Shape Harmonic Absolute Risk Aversion (ISHARA) utility functions to analyze the comparative statics properties of a bargaining model with uncertainty. We identify sufficient and necessary conditions under which an increase in one's degree of risk aversion benefits/hurts one's opponent. We apply our model to analyze the problems of bargaining over insurance contracts and bargaining over incentive contracts.

Keywords Bargaining, the Nash Solution, ISHARA Preference, Risk Aversion

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1 Introduction

In many real-world situations, transactions take place through bargaining. Labour markets in most western economies are characterized by collective agreements negotiated between unions and firms; non-unionized workers' salaries are commonly set by individual negotiation, this being most clearly the case for managerial compensation; firms negotiate over how to split the profits from a joint venture; buyers and sellers bargain over the price of a product; the insurer and his client negotiate over the insurance contract in the insurance market (Kihlstrom and Roth 1982).

It is noticeable that almost all of the bargaining situations mentioned above involve uncertainty (White 2008). Individuals does not know whether an accident will happen when they are bargaining over the insurance contract; the firm and the manager have no idea whether the manager's effort will end up with a good performance when they are deciding the manager's compensation package; producers and retailers are uncertain about the exact demand when they are setting wholesale prices.

Comparing to the well-analyzed situtation of bargaining with deterministic outcome, bargaining with risky outcome is much difficult to study, especially concerning the analysis of comparative statistics. For example, a frequently cited proposition in the deterministic bargaining literature asserts that an increase in one's degree of risk aversion improves the welfare of one's opponent. Intuitively, the subjective possibility of strategically reaching disagreement and its costly consequence makes risk aversion disadvantageous in bargaining (Kannai, 1977; Roth, 1979; Kihlstrom, Roth and Schmeidler, 1981; Sobel, 1981). However, it may fail in the case of risky outcome and riskless disagreement (Roth and Rothblum, 1982), and in the case of risky outcome and risky disagreement (Safra, Zhou and Zilcha, 1990).

The complexity of the analysis of the comparative statics properties of bargaining models with risky outcome and risky disagreement impedes the application of such models. This paper provides a simple method by focusing on the Identical Shape Harmonic Absolute Risk Aversion (ISHARA) utility functions. The ISHARA assumption — equivalent to risk tolerances that are linear in income with identical slope — implies an aggregation property: the sum of the certainty equivalents for the two bargainers is independent of the sharing rule that is used as long as the sharing rule is "efficient". Therefore the model of bargaining over a risky outcome can be reduced to a problem of bargaining over a certainty equivalent — a riskless outcome.

This transformation allows us to disengage two effects regarding an increase in one's degree of risk aversion: the bargaining power effect and the net surplus effect. On the one hand, a more risk-averse bargainer has weak bargaining power and hence his opponent benefits. On the other hand, an increase in one's degree of risk aversion changes the size of the net certainty equivalent — the total certainty equivalent of agreement minus the sum of the certainty equivalents of the two bargainers' disagreements — that the two bargainers are bargaining over. This will benefit (resp. hurts) his opponent if the size is increased (resp. reduced). Consequently, the welfare of one's opponent will be increased as long as an increase in one's degree of risk aversion increases the net certainty equivalent. The welfare of one's opponent will be reduced if an increase in one's degree of risk aversion significantly reduces the net certainty equivalent.

We then apply our model to analyze two situations: bargaining over insurance contract and bargaining over incentive contract.

That the insurance is determined through bargaining between insurer and client is justified if neither of them is small. Kihlstrom and Roth (1982) already studied such a problem with very general utility function. They show that an insurer always benefits as the client becomes more risk averse. However, they only analyze the case of risk-neutral insurer, although they noticed that the assumption of insurer risk neutrality can not be justified in some interesting situations. They argue that subsequent steps require an extension of their results to the case of risk averse insurer. This is exactly the work of this paper. The simple transformation allow us to check easily that their results are still valid in the case of risk averse insurer.

Another application considers the problem of bargaining over incentive contract. Standard principal-agent models always assume that the principal offers a "take-it-or-leave-it" offer. However, as we already argued, it is common that, in real-life situations, both parties hold some bargaining power. Economists start to build models of bargaining over incentive contracts and show that the distribution of bargaining power between principal and agent has real effects (Pitchford 1998, Balkenborg 2001, Schmitz 2005, Demougin and Helm 2006, Demougin and Helm 2009, Dittrich and Städter 2011, Yao 2012). However, those literatures only consider the case with a risk-neutral principal and a wealth-constrained risk neutral agent (or, a risk neutral agent with limited liability). This paper complements this literature by considering bargaining between a risk neutral principal and a risk averse agent à la Holmstrom and Milgrom (1987).

We show that the bargaining model predicts the same incentives and total surplus as in the model where the principal offers "take-it-or-leave-it" offer. However, the principal's preference over the agent's degree of risk aversion is quite different. If the principal holds all the bargaining power, he suffers if the agent becomes more risk averse, as providing incentive becomes more costly and hence the total surplus is reduced. When the contract is determined through bargaining, this result may not hold. This is because an increase in the agent's degree of risk aversion has two effects. On the one hand, an increase in the agent's degree of risk aversion reduces the total surplus; on the other hand, it also reduces the agent's bargaining power. For sufficiently riskless production process, the first effect is dominated by the second one, leading to a higher utility for the principal.

The paper is organized in the following. Section 2 lays out the basic model. Section 3 provides solution. Section 4 applies the besic model the the problems of bargaining over insurance and bargaining over incentive contract. We conclude in section 5.

2 The Nash Bargaining Game

Two bargainers are bargaining over a risky outcome $\tilde{Y}$. Bargainer i has vNM utility function $u_i(w) : [0, \infty) \to \mathbb{R}$, $i = 1, 2$. The bargaining game is defined by a pair (S, d), where $S = \{(Eu_1(s(\tilde{Y})), Eu_2(\tilde{Y} - s(\tilde{Y})))|0 \leq s(Y) \leq Y\}$ is the set of (unanimously agreed) feasible expected utility payoffs to the bargainers, $d = (Eu(\tilde{y}_1), Eu(\tilde{y}_2)) \in S$ is the disagreement point, $s(Y)$ is the risk sharing rule that maps each realized value of $\tilde{Y}$ to bargainer 1's individual share, and $\tilde{y}_i$ is bargainer i's disagreement payoff.

We allow $\tilde{Y}$ and $\tilde{y}_i$s to be degenerated random variables, i.e., riskless variables. If none of $\tilde{Y}$ and $\tilde{y}_i$s is degenerated, we are in the case of risky agreement and risky disagreement; if $\tilde{Y}$ is degenerated, we are in the case of riskless agreement and risky disagreement; if $\tilde{y}_i$s are degenerated, we are in the case of risky agreement and riskless disagreement.

The Nash solution will specify risk-sharing rules $\hat{s}(Y)$, which solves the following problem:

$$\textbf{P1} \ \max_{s(Y)} \Big((Eu_1(s(\tilde{Y})) - Eu_1(\tilde{y}_1)) \cdot (Eu_2(\tilde{Y} - s(\tilde{Y})) - Eu_2(\tilde{y}_2)) \Big),$$

and yields the bargaining outcomes $F_1(S, d) = Eu(\hat{s}(\tilde{Y}))$, $F_2(S, d) = Eu(\tilde{Y} - \hat{s}(\tilde{Y}))$ for bargainer 1 and 2 respectively.

Now assume bargainer 2 becomes more risk-averse, i.e., his utility function becomes $v_2(c)$, with $\frac{-v_2''(c)}{v_2'(c)} > \frac{-u_2''(c)}{u_2'(c)}$. The question that is central to this paper is: will bargainer 1 be better off?

3 Solution with ISHARA utility functions

Problem **P1** concerning risk-sharing rules is not easy to solve. We hence focus on the case of Identical Shape Harmonic Absolute Risk Aversion (ISHARA) utility functions. Following Schulhofer-Wohl (2006), we give the following definition of ISHARA:

Definition 1 *The two bargainers have ISHARA preferences if their utility functions are given by* $u_i(c) = \frac{(c-\theta_i)^{1-\sigma}}{1-\sigma}$, $i = 1, 2$, *where* $\sigma \geq 0$ *is common to both bargainers and* θ_i *is bargainer i's individual parameter.*

Notice that the constant absolute risk aversion is a special case in the limit as σ goes to infinity with θ/σ fixed.

Aggregation Property. It is well known that with ISHARA utility functions, the Pareto frontier in the monetary-equivalent space is a straight line and the monetary value of the joint pie is distribution-free, i.e., the sum of the two bargainers' certainty equivalents is *constant* for any efficient risk sharing rule and *does not* depend on the weights given to the bargainers (see Schulhofer-Wohl 2006 for a proof). We call this property the **aggregation property**. Thus the Nash solution to bargaining with risky outcomes and risky disagreement points can be viewed as the division of a fixed amount of certainty equivalent between two risk-averse bargainers.

Denote C as the total certainty equivalent bargained over by two bargainers, C_1, C_2 as their respective share, and $C_1^d \triangleq u_1^{-1}(Eu_1(\tilde{y}_1)), C_2^d \triangleq u_2^{-1}(Eu_2(\tilde{y}_2))$ as their disagreement payoffs in monetary terms. The net surplus in terms of certainty equivalent is $NC = C - \left(C_1^d + C_2^d\right) > 0$. Henceforth, whenever we say "the size of the pie", we refer to the net surplus NC.

Since the Nash solution is Pareto optimal and satisfies the axiom of independence of irrelevant alternatives, we can restrict our attention to the Pareto frontier which, under this transformation, is given by $S^p = \{(u_1(C_1), u_2(C_2)) \,| C_1 \geq 0, C_2 \geq 0, C_1 + C_2 = C\}$[1]. Because each bargainer should obtain at least his disagreement utility, we can further restrict our attention to $\tilde{S}^p = \{(u_1(C_1), u_2(C_2)) \,| C_1 \geq C_1^d, C_2 \geq C_2^d, C_1 + C_2 = C\}$, which, using the expression of NC, can be rewritten as $\tilde{S}^p = \{\left(u_1(C_1^d + x), u_2(C_2^d + NC - x)\right) |0 \leq x \leq NC\}$. It can be easily proved that there exists a unique Nash solution on $\tilde{S}^P$, and the solution (in the certainty-equivalent space) can be obtained from the following

[1] Independence of irrelevant alternatives means that the solution to the bargaining problem does not change if the utility possibilities set is unfavorably altered such that the disagreement point is unchanged and the original solution remains feasible. That is, if (S, d) and (S', d) are bargaining problems and $S' \subset S$, and the solution of (S, d) also belongs to S', then the two bargaining problems have the same solution.

maximization problem:

$$\mathbf{P2} \max_{0\leq x\leq NC} \left(\left(u_1\left(C_1^d+x\right)-u_1\left(C_1^d\right)\right)\cdot\left(u_2\left(C_2^d+NC-x\right)-u_2\left(C_2^d\right)\right)\right).$$

Thus, we have transformed the bargaining model with risky agreement and risky disagreement into a bargaining model with riskless agreement and riskless disagreement. Denote $w_1(c)=u_1\left(C_1^d+c\right)$ and $w_2(c)=u_2\left(C_2^d+c\right)$. The above bargaining problem can be viewed as two bargainers, whose utility functions are $w_1(c)$ and $w_2(c)$, and who are bargaining over a riskless pie NC, with disagreement payoffs zero.

$$\mathbf{P2'} \max_{0\leq x\leq NC} \left(\left(w_1(x)-w_1(0)\right)\cdot\left(w_2(NC-x)-w_2(0)\right)\right).$$

Denote the solution as $x^{\#}$. The F.O.C. with respect to x gives:

$$w_1'\left(x^{\#}\right)\left[w_2\left(NC-x^{\#}\right)-w_2(0)\right]-w_2'\left(NC-x^{\#}\right)\left[w_1\left(x^{\#}\right)-w_1(0)\right]=0, \quad (1)$$

which, after rearranging, yields:

$$\frac{w_1\left(x^{\#}\right)-w_1(0)}{w_1'\left(x^{\#}\right)}=\frac{w_2\left(NC-x^{\#}\right)-w_2(0)}{w_2'\left(NC-x^{\#}\right)}, \quad (2)$$

i.e., the ratio of each bargainer's net share of the pie in terms of expected utility to marginal utility should be equal.

Now consider the effect of replacing bargainer 2's preference with a more risk-averse utility function v_2. The increase in risk aversion has two effects. First, it reduces the sum of the certainty equivalent. Denote the reduced amount as $\Delta C=C-C^*$, where we use the superscript $*$ to denote the corresponding variables in the new bargaining game between bargainer u_1 and bargainer v_2. Second, it also reduces the disagreement certainty equivalent of bargainer 2. Denote the reduced amount as $\Delta C_2^d=C_2^d-C_2^{d*}$. The reduced amount of the size of the pie (the net surplus) is hence $\Delta NC=NC-NC^*=\Delta C-\Delta C_2^d$. When $\Delta NC>0$, the size of the pie decreases after the replacement; when $\Delta NC<0$, the size of the pie increases after the replacement. The solution x^* solves the following problem:

$$\mathbf{P3} \max_{0\leq x\leq NC^*} \left(\left(w_1(x)-w_1(0)\right)\cdot\left(w_2^*(NC^*-x)-w_2^*(0)\right)\right),$$

where $NC^*=C^*-C_1^d-C_2^{d*}$ is the net surplus in the bargaining game between bargainer u_1 and bargainer v_2, and $w_2^*(c)=v_2\left(C_2^{d*}+c\right)$. The bargaining game can be viewed as two bargainers, whose utility functions are $w_1(c)$ and $w_2^*(c)$, and who are bargaining over a riskless pie NC^*, with disagreement payoffs zero. Similarly, as in solving **P2′**, the F.O.C. yields

$$\frac{w_1(x^*) - w_1(0)}{w_1'(x^*)} = \frac{w_2^*(NC^* - x^*) - w_2^*(0)}{w_2^{*\prime}(NC^* - x^*)}. \tag{3}$$

Bargainer 1 prefers to bargain with bargainer v_2 rather than with bargainer u_2 if $x^* \geq x^{\#}$, which is the case iff

$$w_1'(x^*)[w_2(NC - x^*) - w_2(0)] - w_2'(NC - x^*)[w_1(x^*) - w_1(0)] \leq 0,$$

which, after rearranging, yields

$$\frac{w_2(NC - x^*) - w_2(0)}{w_2'(NC - x^*)} \leq \frac{w_1(x^*) - w_1(0)}{w_1'(x^*)}.$$

Substitute equation (3) into the above inequality, and we get the necessary and sufficient condition of $x^* \geq x^{\#}$:

$$\frac{w_2(NC - x^*) - w_2(0)}{w_2'(NC - x^*)} \leq \frac{w_2^*(NC^* - x^*) - w_2^*(0)}{w_2^{*\prime}(NC^* - x^*)}, \tag{4}$$

which can be rewritten as:

$$\begin{aligned}&[\frac{w_2(NC - x^*) - w_2(0)}{w_2'(NC - x^*)} - \frac{w_2^*(NC - x^*) - w_2^*(0)}{w_2^{*\prime}(NC - x^*)}] \\ &+[\frac{w_2^*(NC - x^*) - w_2^*(0)}{w_2^{*\prime}(NC - x^*)} - \frac{w_2^*(NC^* - x^*) - w_2^*(0)}{w_2^{*\prime}(NC^* - x^*)}] \leq 0.\end{aligned} \tag{5}$$

An increase in one's degree of risk aversion has two effects on one's opponent's welfare. First, because one becomes more risk-averse, one's bargaining power will change. The term in the first square bracket reflects this bargaining power effect, because it keeps the net certainty equivalent unchanged. Second, the net surplus also changes as one becomes more risk-averse. This net surplus effect is reflected by the terms in the second square bracket.

Lemma 1 $\frac{w_2(NC-x^*)-w_2(0)}{w_2'(NC-x^*)} - \frac{w_2^*(NC-x^*)-w_2^*(0)}{w_2^{*\prime}(NC-x^*)} \leq 0.$

Proof. *Denote $\delta = NC - x^*$. The inequality is equivalent to*

$$\int_0^\delta \frac{w_2'(c)}{w_2'(\delta)}dc \leq \int_0^\delta \frac{w_2^{*\prime}(c)}{w_2^{*\prime}(\delta)}dc$$

$$\Leftarrow \frac{w_2'(c)}{w_2'(\delta)} \leq \frac{w_2^{*\prime}(c)}{w_2^{*\prime}(\delta)}, \forall c < \delta$$

$$\Leftrightarrow \frac{w_2'(c)}{w_2^{*\prime}(c)} \leq \frac{w_2'(\delta)}{w_2^{*\prime}(\delta)}, \forall c < \delta,$$

which holds if $\frac{w_2'(c)}{w_2^{*\prime}(c)}$ *is increasing in c.*

$$\frac{\partial}{\partial c}\frac{w_2'(c)}{w_2^{*\prime}(c)} = \frac{w_2''(c)\, w_2^{*\prime}(c) - w_2'(c)\, w_2^{*\prime\prime}(c)}{w_2^{*\prime}(c)^2} > 0,$$

$$\Leftrightarrow -\frac{w_2''(c)}{w_2'(c)} < -\frac{w_2^{*\prime\prime}(c)}{w_2^{*\prime}(c)}.$$

Because bargainer v_2 *is more risk-averse than bargainer* u_2*, we have* $-\frac{w_2''(c)}{w_2'(c)} = -\frac{u_2''\left(C_2^d+c\right)}{u_2'\left(C_2^d+c\right)} < -\frac{v_2''\left(C_2^d+c\right)}{v_2'\left(C_2^d+c\right)}$*. Moreover, our assumption that* $\sigma \geq 0$ *implies* v_2 *exhibits Decreasing Absolute Risk Aversion property, and hence* $-\frac{v_2''\left(C_2^d+c\right)}{v_2'\left(C_2^d+c\right)} < -\frac{v_2''\left(C_2^{d*}+c\right)}{v_2'\left(C_2^{d*}+c\right)} = -\frac{w_2^{*\prime\prime}(c)}{w_2^{*\prime}(c)}$ *due to* $C_2^{d*} < C_2^d$*.* ■

Notice that equation (4) is equivalent to Lemma 1, given that $NC = NC^*$. The above lemma states that an increase in bargainer 2's degree of risk aversion, if it doesn't affect the net bargaining surplus, i.e., $\Delta C = \Delta C_2^d$, will make bargainer 1 better off. This result is consistent with the prevailing predictions on the Nash solution with risk-averse bargainers: risk aversion benefits one's opponent (Kihlstrom, Roth, and Schmeidler, 1981; Roth, 1979, among others). Disagreement has costly consequences, and the desire to avoid the risk of disagreement is reflected in the final bargaining outcome. A more risk-averse bargainer has a stronger desire to avoid such risk, and hence is willing to give up more share during the bargaining in order to facilitate reaching an agreement.

Lemma 2 $\frac{w_2^*(NC-x^*)-w_2^*(0)}{w_2^{*\prime}(NC-x^*)}$ *is increasing in NC.*

Proof. *The result is straightforward by noticing that* $w_2^*(NC - x^*) - w_2^*(0)$ *is increasing in* NC *and that* $w_2^{*\prime}(NC - x^*)$ *is decreasing in* NC*.* ■

Thus, the term in the second square bracket of (5), reflecting the net surplus effect, is negative if $NC^* > NC$. It states an intuitive result: bargainer 1 will be better off as the size of the pie increases.

Combining lemma 1 and lemma 2 yields the following proposition:

Proposition 1 *An increase in one's degree of risk aversion benefits one's opponent if the net certainty equivalent increases. It hurts one's opponent only if the net certainty equivalent decreases significantly, i.e., when it outweighs the opponent's benefit from the increase of relative bargaining power.*

As bargainer 2 becomes more risk-averse, the total certainty equivalent will decrease significantly when the agreement income $\tilde{Y}$ is highly risky. Bargainer 2's total certainty equivalent will decrease significantly when his/her disagreement $\tilde{y}_2$ is highly risky. The net certainty equivalent is more likely to increase when $\tilde{Y}$ is not risky and $\tilde{y}_2$ is highly risky; while it will decrease when $\tilde{Y}$ is highly risky and $\tilde{y}_2$ is not risky. In the case of riskless agreement and risky disagreement, the total certainty equivalent does not change, while bargainer 2's certainty equivalent of disagreement decreases as he/she becomes more risk-averse. Thus the net surplus increases and hence benefits bargainer 1. In the case of risky agreement and riskless disagreement, the net certainty equivalent decreases and bargainer 1 may become worse off if $\tilde{Y}$ is very risky. In the case of risky agreement and risky disagreement, whether the net certainty equivalent increases or decreases depends on the relative riskiness of $\tilde{Y}$ and $\tilde{y}_2$.

Finally, the change in the size of the pie also depends on the relative degree of risk aversion of the two bargainers. If bargainer 1 is much less risk-averse than bargainer 2, then bargainer 1 bears most of the risk. Thus, an increase in bargainer 2's degree of risk aversion would not change the total certainty equivalent too much. In the extreme case where bargainer 1 is risk neutral, the total certainty equivalent remains unchanged. The size of the pie therefore increases as bargainer 2's certainty equivalent of disagreement decreases. Similar arguments tell us that the size of the pie will be reduced if bargainer 2 is much less risk-averse than bargainer 1. We summarize our results in the following proposition:

Proposition 2 *1) With riskless agreement, an increase in a bargainer's degree of risk aversion always increases his/her opponent's welfare.*

2) With risky agreement, an increase in a bargainer's degree of risk aversion may increase or reduce his/her opponent's welfare. An increase in a bargainer's degree of risk aversion is more likely to reduce (resp. increase) his/her opponent's welfare if the agreement is highly (resp. less) risky, the bargainer's disagreement is less (resp. highly) risky, and the bargainer is much less (resp. more) risk-averse than his opponent.

We illustrate the above proposition with the following example.

Example 1 *Consider the case where two bargainers have CARA utility function* $u_i(c) = \frac{1-\exp(-r_i c)}{r_i}$, $i = 1, 2$. *Assume,* $\tilde{Y} \sim N(\mu, \sigma^2)$, $\tilde{y}_i \sim N(\mu_i, \sigma_i^2)$. *The specific assumption allow us to write* $C = \mu - \frac{R}{2}\sigma^2$, $C_1^d = \mu_1 - \frac{r_1}{2}\sigma_1^2$ *and* $C_2^d = \mu_2 - \frac{r_2}{2}\sigma_2^2$. *The net certainty equivlant is given by* $NC = C - C_1^d - C_2^d$.

An increases in r_2 *will benefit (resp. hurts) bargainer 1 if* $f(r_2, NC) = \frac{u_2\left(C_2^d+NC-x\right)-u_2\left(C_2^d\right)}{u_2'\left(C_2^d+NC-x\right)}$ *is increasing (resp. decreasing) in* r_2. *Denote* $\delta = NC - x$.

$$\begin{aligned}
\frac{d}{dr_2} f(r_2, NC) &= \frac{\partial}{\partial r_2} f(r_2, NC) + \frac{\partial}{\partial NC} f(r_2, NC) \frac{\partial NC}{\partial r_2} \\
&= \frac{1}{r_2}\left(1 + r_2 \delta e^{r_2\delta} - e^{r_2\delta}\right) + e^{r_2\delta}\left[\frac{1}{2}\left(\sigma_2^2 - \frac{\partial R}{\partial r_2}\sigma^2\right)\right] \\
&= \frac{1}{r_2}\left(1 + r_2 \delta e^{r_2\delta} - e^{r_2\delta}\right) + e^{r_2\delta}\left[\frac{1}{2}\left(\sigma_2^2 - \frac{r_1^2}{(r_1+r_2)^2}\sigma^2\right)\right].
\end{aligned}$$

It is easy to prove that $1 + r_2\delta e^{r_2\delta} - e^{r_2\delta} > 0$. *Therefore, we have* $\frac{d}{dr_2} f(r_2, NC) > 0$ *when* $\sigma_2^2 - \frac{r_1^2}{(r_1+r_2)^2}\sigma^2 > 0$, *which is more likely to be the case if* σ_2 *is large,* σ^2 *is small and that* r_2 *is much larger than* r_1.

That $\frac{d}{dr_2} f(r_2, NC) < 0$ *occurs only if* $\sigma_2^2 - \frac{r_1^2}{(r_1+r_2)^2}\sigma^2 < 0$. *Consider the case with riskless disagreement where* $\sigma_2^2 = 0$. $\frac{d}{dr_2} f(r_2, NC) < 0$ *will be the case if* $\sigma^2 > \hat{\sigma}^2$, *with* $\hat{\sigma}^2 = \frac{2(1+r_2/r_1)^2\left(1+r_2\delta e^{r_2\delta} - e^{r_2\delta}\right)}{r_2 e^{r_2\delta}}$. *Notice that* $\hat{\sigma}^2$ *is increasing in* r_2/r_1, *which means that* $\frac{d}{dr_2} f(r_2, NC) < 0$ *is easier to be satisfied if* r_1 *is much larger than* r_2.

4 Applications

4.1 Bargaining Over Insurance Contract

In this section, we apply our model to study the insurance contracts reached through bargaining. The model has already studied by Kihlstrom and Roth (1982). However, they only consider the case with risk-neutral insurer. The assumption of risk-neutral insurer is appropriate if the insurer insueres many risks independent of that being analyzed and hence diversify these risks. In other interesting situations, however, the assumption of insurer risk neutraility can not be justified (Kihlstrom and Roth 1982). In this section we apply our basic model and provide a simple method to re-consider this problem and, specially, to study the situation with risk-averse insurer.

Consider a sitution with two individuals: a client and an insurer. Both the insurer and the client are risk averse and have ISHARA utility functions: $u_i(c) = \frac{(c-\theta_i)^{1-\sigma}}{1-\sigma}$, $i = I, C$, where I, C represent insurer and client.

The client faces a possible financial loss. His wealth is a binary random variable:

$$\tilde{w}_C = \begin{cases} w_C > 0 \text{ with probability } v \\ w_C - L > 0 \text{ with probability } 1 - v \end{cases}.$$

The insurer's wealth is w_I and he is not faced with the possibility of any exogenous losses. Assume the insurer has sufficient wealth so that he will have the resources to provide complete coverage in any cases.

The insurer agrees to insure the client and bear some of the burden of the client's loss in the event it arises. His wealth is

$$x_{Il} = w_I - A$$

if the loss occurs and

$$x_{In} = w_I + p$$

if the loss does not occur.

With this insurance contract in force, the client's wealth is

$$x_{Cl} = w_C - L + A$$

if the loss occurs and

$$x_{Cn} = w_C - p.$$

if the loss does not occur.

Let's first consider the case of a risk-neutral insurer and a risk averse client. In a competitive insurance market, the client is completely insured and the insurer's expected wealth is equal to w_I. The competitive equilibrium contract (A, p) is unchanged by an increase in the client's risk aversion, and is determined by the following two equations

$$L - A = p,$$

$$(1 - v)\, p - vA = 0.$$

Now we assume that the insurance contracts are reached through Nash bargaining. Pareto optimality of the Nash solution requires that the risk-neutral bear all the risks. That is, the client is completely insured. The total surplus C is hence given by

$$C = w_C + w_L - (1 - v)\, L,$$

regardless of the degree of the risk aversion of the client.

The client's disagreement payoffs is $C_2^d \triangleq u_C^{-1}(Eu_C(\tilde{w}_C))$ in monetary terms. As the client becomes more risk averse, C_2^d decreases. Thus, an increase in the client's risk averse increases the net certainty equivalent that the insurer and the client are bargaining over, because it does not change the total certainty equivalent but reduces the client's disagreement certainty equivalent. It follows from proposition 1 that the insurer is better off, which is the result of theorem 4.1 in Kihlstrom and Roth (1982).

Moreover, that the insurer is better off means $(1-v)\,p - vA$ increases. Because the client is completely insured, we have $L - A = p$. It follows immediately that p increases and A decreases: a more risk averse client pays a higher premium and receives less coverage of his potential loss.

Now we turn to the case where both of them are risk averse. The total certainty equilent $C = w^{-1}(Ew(\tilde{w}_C + w_I))$, where w is the representative's utility function, with $w = \frac{(c-\theta_C-\theta_I)^{1-\sigma}}{1-\sigma}$. The disagreement payoff of the client is equal to $C_2^d \triangleq u_C^{-1}(Eu_C(\tilde{w}_C))$, where $u_C = \frac{(c-\theta_C)^{1-\sigma}}{1-\sigma}$.

Lemma 3 *The net certainty equivalent increases as the client becomes more risk averse.*

Proof. We need to prove

$$\frac{\partial NC}{\partial \theta_C} = \frac{\partial C}{\partial \theta_C} - \frac{\partial C_2^d}{\partial \theta_C} \geq 0.$$

Using the specific formula of u_C and w, we know

$$C_2^d = \left[E\left(\tilde{w}_C - \theta_C\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} + \theta_C$$

and

$$C = \left[E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} + \theta_I + \theta_C.$$

Notice that the insurer have sufficient wealth and hence $w_I - \theta_I > 0$. The above two equations imply that we will be done if we can prove $\frac{\partial^2 C}{\partial \theta_C \partial \theta_I} \leq 0$. From above equation, we have

$$\frac{\partial C}{\partial \theta_C} = -\left[E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma}\right]^{\frac{\sigma}{1-\sigma}} E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{-\sigma} + 1,$$

and therefore

$$\frac{\partial^2 C}{\partial \theta_C \partial \theta_I} = \sigma\{\left[E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma}\right]^{\frac{2\sigma-1}{1-\sigma}} \left(E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{-\sigma}\right)^2$$
$$- \left[E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma}\right]^{\frac{\sigma}{1-\sigma}} E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{-\sigma-1}\}$$
$$= \sigma\left[E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma}\right]^{\frac{2\sigma-1}{1-\sigma}} \{\left(E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{-\sigma}\right)^2$$
$$- E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{1-\sigma} E\left(\tilde{w}_C + w_I - \theta_I - \theta_C\right)^{-\sigma-1}\}$$
$$\leq 0,$$

where the last inequality holds as a direct application of Cauchy-Schwarz inequality. ■

Notice that an increase in the client's degree of risk aversion reduces both the total surplus and the client's certainty equivalent of disagreement. In the case of no disagreement, the certainty equivalent C_2^d is reduced significantly because the client alone bear all the risks. In the case of agreement, however, the total certainty equivalent C is reduced only slightly because the insurer share some risks. As a result, the reduced amount of C is much less than the reduced amount of C_2^d and therefore the net certainty equivalent increases.

The above lemma together with proposition 1 immediately gives the following proposition

Proposition 3 *The insurer, whether he is risk-neutral or risk averse, benefits as the client becomes more risk averse.*

Calculating the bargained Insurance Contract. Now we illustrate how we can calculate the bargained contract (A, p) from the transformed problem. First, from equation (1), we can calculate the exact net certainty equivalent that the insurer gets $(x^{\#})$ and the client gets $(NC - x^{\#})$. Then, the contract (A, p) can be calculated from the definition of certainty equivalent, which are given by the following two equations:

$$vu_C\left(w_C - p\right) + (1-v)\, u_C\left(w_C - L + A\right) = u_C\left(C_2^d + NC - x^{\#}\right),$$

$$vu_I\left(w_I + p\right) + (1-v)\, u_I\left(w_I - A\right) = u_I\left(w_I + x^{\#}\right).$$

4.2 Bargaining Over Incentive Contract

Standard principal-agent models often assume that the principal offers "take-it-or-leave-it" contracts to the agent. Consequently, the principal obtains all the surplus of the transaction. A direct result is that the principal suffers from an increase in the degree of the agent's risk aversion, because the cost of providing higher incentive increases as the agent becomes more risk averse.

However, in many real-world situations, both parties hold some bargaining power and hence the contracting involves bargaining. For example, many labour market situations are characterized by bargaining between workers and firms (Demougin and Helm 2006). We will prove in this section that bargaining will change significantly the property of comparative statistics. Especially, we will show that the principal may benefit if the agent becomes more risk averse.

Consider the case where a risk neutral principal bargaining with a risk averse agent over an incentive contract. The principal hires the agent to produce output. The agent has CARA utility function with absolute risk averse coefficient r: $u(x) = \frac{1-\exp(-rx)}{r}$. The agent can exert costly effort to increase output. The output is

$$y = e + \varepsilon,$$

where $\varepsilon \sim N(0, \sigma^2)$, with σ^2 representing the riskiness involved in the production process, while e representing the effort exerted by the agent. The effort cost is $c(e) = \frac{e^2}{2}$.

Contract. The effort is not observable. The only observable and contractible variable is the output y. Assume the contract that the two parties are bargaining over is linear:

$$w = w_0 + \alpha y,$$

where w_0 is the fixed salary and α is the power of incentive.

The timing is follows. First, the two parties engage in Nash bargaining process and bargain over the contract (w_0, α). If no agreement is reached, the game is over and both of them get nothing. If a contract is signed, then the agent chooses his effort. Finally, output is realized and the contract is excuted.

Given the contract, the agent choose e to maximize his certainty equivalent

$$C_A = w_0 + \alpha e - \frac{r\alpha^2\sigma^2}{2} - \frac{e^2}{2}.$$

F.O.C with respect to e gives the following incentive compatible condition

$$IC : e = \alpha.$$

For contract (w_0, α), the total certainty equivalent of the principal and the agent is

$$C = e - \frac{e^2}{2} - \frac{r\alpha^2\sigma^2}{2}.$$

Subsitute the IC condition into the expression of C and C_A, we obtain $C(\alpha) = \alpha - \frac{(1+r\sigma^2)\alpha^2}{2}$ as a function of α and $C_A(\alpha, w_0) = w_0 + \frac{(1-r\sigma^2)\alpha^2}{2}$ as a function of α and w_0. Notice that both the principal and the agent get nothing if no agreement is reached. Hence, the Nash Bargainig solution is given by the following problem

$$\max_{w_0,\alpha} (C(\alpha) - C_A(\alpha, w_0)) u (C_A(\alpha, w_0)).$$

The solution of Nash bargaining implies that two parties will choose α to maximize the total certainty equivalent $C(\alpha)$. Otherwise, suppose the solution is (w_0', α'), while there exists α^* such that $C(\alpha^*) > C(\alpha')$. Then one can choose a proper w_0^* such that $C_A(\alpha', w_0') = C_A(\alpha^*, w_0^*)$. Obviously, (w_0^*, α^*) gives a higher value of $(C(\alpha) - C_A(\alpha, w_0)) u (C_A(\alpha, w_0))$, contrading that (w_0', α') is the Nash solution.

The first order condition of $C'(\alpha) = 0$ immediately gives

$$\alpha^* = \frac{1}{1 + r\sigma^2}.$$

The net certainty equivalent is equal to the total surplus and is given by

$$\begin{aligned} NC = C &= \frac{1}{1+r\sigma^2} - \frac{1}{2}\left(\frac{1}{1+r\sigma^2}\right)^2 - \frac{r\sigma^2}{2}\left(\frac{1}{1+r\sigma^2}\right)^2 \\ &= \frac{1}{2}\frac{1}{1+r\sigma^2}. \end{aligned}$$

Proposition 4 *Comparing to the case where the principal has all the bargaining power, the bargaining model predicts the same power of incentive (α) and hence the same total surplus.*

The existing literatures considering bargaining contract between principal and agent often assume risk neutral agent with limited liability (Pitchford 1998, Balkenborg 2001, Demougin and Helm 2006). The main result is that the bargaining model and the take-it-or-leave-it model prodict different incentives. The above proposition is in contrast with this result and provides an example where bargaining does not have real effect. However, as we will show immediately, the principal's preference over the agent's degree of risk aversion is quite different from the take-it-or-leave-it model.

The above analysis show that the bargaining model can be viewed as if the principal and the agent are bargaining over a total surplus $C = \frac{1}{2}\frac{1}{1+r\sigma^2}$, with outside option normalized to zero. Hence, we can rewrite the problem as:

$$\max_x xu(C-x)$$

The first order condition gives:

$$\frac{1-\exp(-r(C-x))}{r} - x\exp(-r(C-x)) = 0,$$

from which we get

$$x = \frac{1}{r}(\exp(r(C-x)) - 1).$$

where $C = \frac{1}{2}\frac{1}{1+r\sigma^2}$. Define $L = \frac{1}{r}(\exp(r(C-x)) - 1)$, then we know $\frac{\partial x}{\partial r} \geq 0$ iff $\frac{dL}{dr} \geq 0$.

$$\begin{aligned}\frac{dL}{dr} &= \frac{\partial L}{\partial r} + \frac{\partial L}{\partial C}\frac{\partial C}{\partial r} \\ &= \frac{1}{r^2}[1 + r(C-x)\exp(r(C-x)) - \exp(r(C-x))] \\ &- \exp(r(C-x))\left[\frac{1}{2}\frac{\sigma^2}{(1+r\sigma^2)^2}\right]\end{aligned}$$

Obviously, for σ^2 close to zero, $\frac{dL}{dr}$ is strictly positive. Hence, the principal benefit from an increase in the agent's risk aversion.

Proposition 5 *The principal may benefit or hurt by an increase in the agent's degree of risk aversion. Specially, he benefits from an increase in the agent's degree of risk aversion if the production process is sufficiently riskless.*

An increase in the agent's degree of risk aversion has two effects. On the one hand, an increase in the agent's degree of risk aversion reduces the total surplus, which hurts the principal. On the other hand, the agent's bargaining power becomes weaker as he becomes more risk averse, which benefit the principal. For sufficiently riskless production process, the first effect is dominated by the second one, leading to a higher utility for the principal.

Figure 1 illustrate how the principal's payoff x varies with the agent's degree of risk aversion, given different σ. We can see that for small value of σ, the principal's payoff is increasing in the agent's degree of risk aversion; for large value of σ, the principal's payoff is decreasing in the agent's degree of risk aversion; for middle value of σ, the principal's payff is first increasing and then decreasing in the agent's degree of risk aversion.

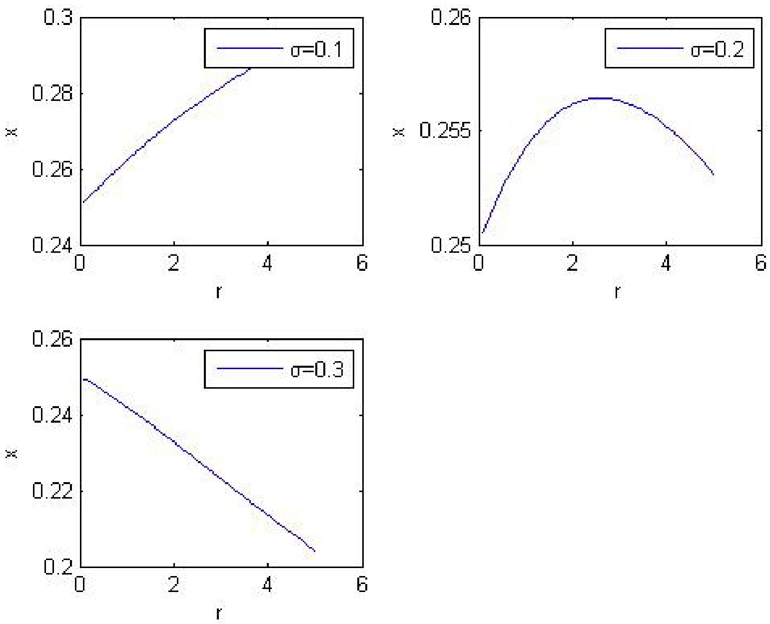

Figure 1

5 Conclusion

This paper builds a simple Nash bargaining model with uncertainty. Especially we identify the two effects of a change of one bargainer's degree of risk aversion: the bargaining power effect and the net surplus effect. An increase in one bargainer's degree of risk aversion reduces his bargaining power, while it also changes the net surplus at the same time. Whether this benefits his opponent depends on which effect dominates.

The simplicity of our model allow us to apply it in many situations. In an application to bargaining over insurance contract, we show that Kihlstrom and Roth (1982)'s result, which states an risk-neutral insurer is better off if the insuree becomes more risk averse, is also robust for the case of risk averse insurer. Applying our model in the situtation of bargaining over incentive contract à la Holmstrom and Milgrom (1987), we show that the principal may benefit as the agent becomes more risk averse, contrasting with the prediction of the take-it-or-leave-it model.

The symmetric Nash bargaining model that we have discussed in the paper can be easily extended to the case of asymmetric Nash bargaining. The asymmetric Nash solution will specify risk-sharing rules $\hat{s}(Y)$, which solves the following problem:

$$\max_{s(Y)} \left((Eu_1(s(\tilde{Y})) - Eu_1(\tilde{y}_1))^{\alpha} \cdot (Eu_2(\tilde{Y} - s(\tilde{Y})) - Eu_2(\tilde{y}_2)) \right)^{1-\alpha},$$

where the parameter α measures the bargaining power of each bargainer. A higher α means that the bargainer 1 has a higher bargaining power. A nature question is how bargainer 1's bargaining power α affect the property of comparative statistics. As α increases, the net surplus effect caused by an increase in bargainer 2's risk aversion becomes more relevant, while the bargaining power effect caused by an increase in bargainer 2's risk aversion becomes less relevant.

We believe our model has many other applications, because many real-world bargaining games involve uncertainty. For example, our model can be applied in the situations where agents form small groups (marriage, partnership...) for the purpose of risk sharing. In India, families nd suitable men for their daughters from distant villages to reduce correlation in climatic and production shocks. A primary concern is about the the compositions of such risk-sharing partnerships, i.e., whether agents in the group have similar or disimilar risk preference. An important issue in this literature is the conflict between theory and empirical/experimental evidences. Theoretical matching models predict negative assortative matching (Chiappori and Reny 2006; Legros and Newman 2007; Schulhofer-Wohl 2006). This result is, however, not consistent with empirical and experimental literature (Lam 1988; Charles and Hurst 2003; Di Cagno et al 2012).

One possibility to resolve this conflict is to relax the assumption that the risk sharing rule is determined by competitive market in the theoretical models. Instead, one can assume agents share their joint risky income through Nash bargaining. If we can show that, under some conditions, agents suffers if his partner becomes more risk averse. Then, the resulting matching will be possitive assortative. The most risk averse agent will propose to the most risk averse partner, who is happy to accept the offer. As a result, agents in the group have similar preference, which is consistent with empirical and experimental evidence.

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