Online Auctions with Competitive Sellers

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September, 2009

Abstract

Online auctions are different from traditional ones in two important features: the competition among many sellers and the effects of cross-group externalities on buyers and sellers. This paper considers both these two features to investigate how the sellers set their optimal reserve prices and the buyers bid for winning a good in online auctions when there are competitive sellers, as well as their expected returns. We focus on the following four cases. The first two include the cases of competitive sellers with buyers' multiauctioning, and single-auctioning when all have the common reserve price with the same private value; the last two include cases of collusion and noncollusion among sellers when they have different private values. In the multiauctioning case, surprisingly, the identical reserve price may increase as the number of sellers become bigger, while is decreasing in the number of buyers. By contrast, in the single-auctioning case, the identical reserve price is the same as that in one-seller case, and the bid for each potential buyer is strictly decreasing in the number of sellers. In both cases, the auctioneer and bidder externalities exist and can be positive. Furthermore, in noncollusion case, the equilibrium is a sequential one, where each seller sets his reserve price based on his private value, which makes up a ranking of reserve prices, and leads to the sequential bidding strategy taken by each bidding buyer. But in collusion case, the results are quite similar to the multiauctioning case, expect that the common reserve price is higher. By simple simulation, our main conclusions are demonstrated and supported to a great extent.

Keywords: Online auctions, Competitive sellers, Multiauctioning, Reserve prices, Bidding strategy

JEL Classification: D44, C78, D82

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1 Introduction

In the last decade, the online-auction industry developed drastically, and the business model of auctions on the internet has turned to be very successful, which has been prevalently applied to the area of electronic business, including B2B, B2C, C2C and G2B. The top online-auction sites, such as eBay, Yahoo, uBid, QXL and Overstock, have winned large amount of turnovers and profits. For instance, the turnovers of eBay in 2003 to 2005 are 2.17, 2.90 and 3.00 billion dollars, and just in the second season in 2008, its' turnover achieved 1.46 billion dollars, though it does have other kinds of business, say fixed-price sales.

During the development process of online auctions, quite a lot of fascinating new features other than the ones in traditional auctions gradually occur, which has been summarized in Lucking-Reiley (2000), such as the variety of auctioned goods, duration of auctions, dynamic auctions, some special bidding behaviors, etc. However, Lucking-Reiley hardly attach importance to two most extraordinary unique features of onine auctions: competition among sellers, and the effects of the online auction platform on the behaviors of both selles and bidders (buyers) in terms of design of auction mechanism and the charge structure. From the perspective of two-sided market, the latter feature is substantially induced by the cross-group externalities in the online auctions, including the externalities of sellers on buyers, which can be called "auctioneer externalities", and the externalities of buyers on sellers, which called "bidder externalities". Such perspective is preliminarily supported by Park (2001, 2002).

We incorporate such two important features into our online auction models, which makes our paper be directly correlated with two streams of literatures. The first is the research on multi-unit (or multi-object) auctions since there are also many goods to be auctioned. There are two types of multi-unit auctions: simultaneous auctions and sequential ones. For the former, there are also two kinds: discriminatory and uniform-price. Vickrey (1961) firstly proposed the idea of uniform-price multi-unit auctions. Weber (1983) and Milgrom and Weber (2000) provide a first overview of multi-unit auctions with bidders whose demands are only one unit. For the cases of multi-unit demand, the bidders are usually supposed to be homogeneous (Black and de Meza, 1992; Katzman, 1999) or symmetric when their private values are interdependent (Perry and Reny, 2001, 2002; Ausubel, 2004). The later literatures investigate the influences on the multi-unit auctions of the heterogeneity of bidders, including different participation costs, incomplete information in common-value auctions, and different demands (Engelbrecht-Wiggans and Weber, 1993; von der Fehr, 1994; Krishna and Rosenthal, 1996; Rosenthal and Wang, 1996; Branco, 1997; Burguet and Sakovics, 1997; Menezes and Monteiro, 2004; Vincent and Chanel, 2007). Recently, some papers discuss the multi-objects auctions with heterogeneous items, including the design of a sealed-bid auction procedure (Dasgupta and Maskin, 2000) and a proxy package auction procedure (Ausubel and Milgrom, 2002). Meanwhile, some papers begin to examine the effects of variable supply on the efficiency of multi-unit auctions and compare that in uniform-price and discriminatory auctions (Damianov and Becker, 2008), though previous fixed-supply literatures are in favor of discriminatory auctions (Back and Zender, 1993; Wang and Zender, 2002; Nyborg and Strebulaev, 2004).

Another important concern of multi-unit auctions lies in the sequential ones where the goods are allowed to auction in sequence. The study in this area pays much attention to the price trend induced by bidders' bidding strategies. Related Theories establish that the sequential auctions of identical items will result in, on average, identical (Weber, 1983) or rising prices (Milgrom and Weber, 2000). On the contrary, many empirical evidences indicate that the prices in multi-item sequential auctions are decreasing, primarily including the auctions of condominium units (Ashenfelter and Genesove, 1992), wine (McAfee and Vincent, 1993; Ginsburgh, 1998), jewellery (Chanel and Vincent, 1996), works of art (Pesando and Shum 1996; Beggs and Graddy, 1997), and many other goods.

The second stream is the literature which captures some of the characteristics of online auctions, including reauctioning, collusion among bidders, and very importantly, the effects of online auction house (or platform). For one-seller case, resale situation without auction house has been discussed by a series of papers (McAfee and Vincent, 1997; Horstmann and LaCasse, 1997; Zheng, 2002). However, only in the latest studies, the effects of auction house as a mediator of transactions on the online auctions which can resale are examined (Matros, Zapechelnyuk, 2008a; Matros, Zapechelnyuk, 2008b). At the same time, the buyers who enter the online auctions are empirically observed to reatively easily achieve collusion, for too many online auction platforms utilize English auction format. For example, in the survey of Lucking-Reiley (2000), 121 of the 142 surveyed online auction sites use English ascending-price auctions. By empirical evidences, there are three main collusion strategies taken by buyers: jump bidding (Sherstyuk, 2002; Brusco and Lopomo, 2002; Kwasnica and Sherstyuk, 2007), sniping (Ockenfels and Roth, 2002, 2006; Wintr, 2008), and withholding bid. Furthermore, the effects of the online auction house on the behaviors of both sellers and buyers have been attached importance by Matros and Zapechelnyuk (2008a, 2008b). They analyze from a special perspective of designing the optimal charge schedule of an online auction house, and even of designing the optimal auction mechanism. However, they ignore the cross-group externalities on both transaction sides, which may be the key for the online auction house to charge and influence the behaviors of buyers and sellers from the perspective of two-sided market.

Surprisingly, all the above related literatures ignore these two features. One might argue that the auctions with competitive sellers may be equivalent to the multi-unit ones with only one seller. however, they are quite different from each other in at least three aspects. The first is the strategic design of each seller's reserve price induced by their competition. In multi-unit auctions, either traditional or online, there is only one auctioneer, so most literatures set his reserve price to be zero, or equivalently, ignore reserve price problem. Even some studies consider the problem in sequential auctions, their logics are lack of concern for the effects of sellers' competition. By contrast, each seller will definitely design his reserve price by considering the other ones' strategies when they auction one unit homogeneous good. The second is the multi-auction access and reserve price effects on each buyer's bidding strategy. By the practice of online auctions, each buyer can virtually enter all available auctions for homogenous goods at the same time. So it can be equivalent for him to participate in sequential auctions. Obviously, the strategic design of the sellers' reserve prices have remarkable impacts on his bidding behavior, which is almost not the question in multi-unit auctions. The last but quite important, is the effects of cross-group externalities on the behaviors of both sides. Since the online auction market is a kind of two-sided market, there should be the cross-group externalities on both buyers and sellers. intuitively, on the one hand, as the number of sellers auctioning homogenous goods increases, the probability of purchasing a desired good for each buyer become bigger, which results in the improvement of his expected gain. On the other hand, with more buyers, each one will bid more aggressively and naturally enhance the expected revenue of each seller. However, in mult-unit auctions, there is only one seller, the auctioneer externalities don't exist for each buyer. Based on the evidences Park (2001, 2002) provided, such cross-group externalities may be one of the key factors to attract millions of people to participate in the online auction sites and also the important reason of the rapid development of online auction industry.

Inspired by these differences and the perspective of two-sided market, we offer a distinct analytical framework for online auctions by embedding the competitive sellers and the effects of both auctioneer and bidder externalities in our SIPV auction models. To our knowledge, we are the first to explore the online auction problems by considering competitive sellers. In this paper, we discuss two typical categories of online auction cases: competitive sellers with identical private values and with different ones.

When all sellers have identical private values, each of them sets a common optimal reserve price. If all buyers are allowed to bid in all simultaneous auctions, which is called "multiauction", the common reserve price turn to be lower as the number of potential buyers increases, while may be higher as the number of sellers become larger, which are quite counterintuitive. Meanwhile, the total expected return of each seller strictly decreases when the bidding likelihood ratio of the buyer whose private value is identical to the common reserve price, λ_M^{BLR}, is high enough. However, some lower end buyers may bid more aggressively than they do when there is only one seller, though the expected gain of each buyer who would like to bid in at least one auction strictly increases in the number of sellers when λ_M^{BLR} is high enough. Analogously, if all buyers can only single-auction, namely, can bid in only one auction, the common equilibrium reserve price is the same as that in one-seller case. However, surprisingly, the total expected return of each seller may decrease as the number of them becomes larger and strictly increase when the number of buyers grows only when some conditions are satisfied rather than without any constraint.

When all sellers have different private values, if the information is symmetric for them about their private values, we establish that two reasonble equilibria of noncollusion and collusion among sellers may exist. In noncollusion equilibrium, the seller with higher private value sets higher reserve price. But the design of higher reserve price depends on all of the lower reserve prices. Thus, each buyer who can bid in at least two auctions may take a sequentially descending bidding strategy from the auction with the lowest reserve price to the one with the highest if his private value is in the quite low end, while take a sequentially ascending one if his private value is in the contrary end. Such possible results are quite different from sequential multi-unit auctions where bidders take monotone bidding strategy, though the noncollusion equilibrium is substantially a sequential one. By contrast, the sellers collude with each other and set a common reserve price which is higher than that in the case that each seller has identical private value and buyers can multiauction. If the information is asymmetric for the sellers, all of them can just estimate the order of their own private values, and normally set different reserve prices, which corresponds to a noncollusion equilibrium. But such estimations make the difference of the lowest and highest reserve prices diminish and the difference between each pair of adjacent ones on average as well. Of course, since all buyers can observe the reserve prices, their bidding strategies are the same as that in symmetric information case.

Fortunately, and surely to some extent, in multiauctioning and single-auctioning cases in the first category, we can clearly find out the endogenous auctioneer and bidder externalities. Such two externalities may be positive in less restrictive cases than the ones we condition. Although these two externalities are not so explicit in the second category of online auctions, based on our related conclusions, they may be also positive with some constraints that we don't propose yet in this paper. By simple simulating and comparing four cases: one seller, multiauctioning with common reserve price, collusion and noncollusion, most of our conclusions are demonstrated and explicitly supported to some extent, especially the positivity of both auctioneer and bidder externalities.

The remainder of the paper proceeds as follows. Section 2 depicts the primary features of online auctions. Section 3 builds up the basic model of online auctions with competition among sellers. Section 4 discusses the online auctions with sellers' identical private values, including the cases of multiauctioning and single-auctionming of buyers. The online auctions with sellers' different private values are investigated in section 5, including noncollusion case, where may exist a sequential equilibrium, and collusion one. Section 6 demonstrates our main conclusions and preliminarily supports some of our conjectures about buyers' bidding strategies. Section 7 concludes and proposes some important open questions.

2 The Features of Online Auctions

Online auctions[1] have proved to be particularly popular during the last decade for industrial procurement (B2B, e.g., Alibaba in China), consumer purchase (B2C and C2C, e.g. eBay). Online auctions have many special features which are advantagoues to traditional auctions.

Firstly, onlines auctions permit almost all kinds of goods to be traded, such as fast moving consumer goods, electronic goods, computer goods, clothes, books and even toys. However, all these examples are almost infeasible in traditional auctions.

Secondly, for each good, they are often a quantity of sellers to compete with one another. That means that if you are a buyer and you have lost an auction for an iphone II provided by seller A, you do get chances to bid for the same ones in other sellers' acutions, which increases your winning probability. For instance, On April, 2009, there are at least 83 sellers on eBay and at least 22 sellers on Overstock auctioning American Eagle Silver Dollar.

Thirdly, an specific online auction may usually last for a period of time, e.g., 1 day, 3 days, 5 days, 7 days and 10 days. 7-day duration is mostly applied.

Fourthly, once a seller fails to sell his good, he can reauction it for many times, and theoretically infinte times, even in some auction sites reauction entails extra cost. It implies that online auctions may be dynamic.

Fifthly, since there are usually more than one seller for each good, sellers may have incentive to collusively set their reserve prices when the population of sellers is small enough, which is impossible in traditional auctions.

Finally and quite importantly, at most cases, there must be online auction houses or platfomrs to provide online auction services,[2] and the behaviors of the online auction platforms can have great effects on the returns of both buyers and sellers for the online auction market is a two-sided one. Obviously, if an online auction platfom have more sellers, and subsequently more goods for auctioning, there will be more buyers willing to log in it, and vise versa. Such effects are called positive cross-group externalities and are the reason why eBay are quite famous and profitable throughout the world. As we have known, the online auction platform can change the numbers of both buyers and sellers by adjusting its related fees, and of course can have effects on sellers' design of reserve prices and the buyers' bidding strategies. According to Luking-Reiley (2000), the fees an online auction platform charges may iinclude registration fee, listing fee, feature category and showcase fee, commission or closing fee. At most cases, sellers are charged listing fee and closing fee, buyers are charged nothing (Matros, Zapechelnyuk, 2008a). For instance, the latest fee rates of top 5 auction site Overstock as of August 3rd in 2009 primarily include listing upgrades fees (optional promotional serveices fees), insertion fees, reserve fees, and closing fees (see talbe 1).

[1] There are many similar terms, such as "internet auctions", "web auctions", "network auctions".

[2] Sashi and O'Leary (2002) provides the B2B online auction platform's operating mechanism.

Table 1: Overstock auction fees on August, 2009

Insertion Fees			
Starting Price Range	Fee	Starting Price Range	Fee
$0.01 - $0.99	$0.10	$50.00 - $199.00	$1.55
$1.00 - $9.99	$0.20	$200.00 - $499.99	$2.35
$10.00 - $24.99	$0.40	$500 and up	$3.15
$25.00 - $49.99	$0.75	—	—
Reserve Fees		Closing Fees	
Reserve Price Range	Fee	Closing Amount	Fee Percentage
$0.01 - $49.99	$0.66	$0.01 - $25	3%
$50.00 - $199.99	$1.40	$25.01 - $1,000	$0.75+2% of the balance over $25.00
$200.00 and up	1% of Reserve Price (Max Fee $70)	Over $1,000	$20.25+1% of the balance over $1000

In terms of economic sense, the second, fourth and last features are quite important and even unique for us to understand online auctions. The fourth and last features have been partially investigated by Matros and Zapechelnyuk (2008a, 2008b), hence we just focus on the second one and part of the last one. .

3 Basic Model

We formalize the second feature and part of the last feature (the cross-group externalities) of online auctions in our model by considering online auctions with n buyers and m sellers who have homogenous goods for sales. We require $n >> m$, $m \geq 2$, which is the key for our main conclusions. All buyers (or bidders) have private values for the goods, i.i.d. random variables with a distribution function F on interval $[\underline{v}, \overline{v}]$. We assume that F is differentiable and has positive density f. Additionally, assume that F satisfy Myerson (1981)'s regular condition, namely, $v - \frac{1-F(v)}{f(v)}$ is strictly increasing on $[\underline{v}, \overline{v}]$. Without loss of generality, assume every agent knows his own private value. For simplicity, suppose all agents are risk neutral. Namely, our model is a SIPV model, expect that there are more than one seller.

Consistent with the standard setting of traditional auctions, all sellers auction their goods only one time, and they also announce their reserve prices to the potential buyers. For getting rid of the effects of the online auction platform, we assume the platform provides two main auction formats: Dutch (high bid) and English, and don't charge buyers and sellers (or just requires sellers to pay the listing fees).[3]

[3] In the theory of two-sded market, the online auction can make profit by charging his advertisers, usually manufacturers, the advertisment fees though it may charge the sellers and buyers nothing. In fact, Taobao in China do take such strategy.

For convenience, we define "the bidding likelihood ratio" (BLR) for buyer whose private value is v_i as

$$\lambda_i^{BLR}(v_i) \equiv \frac{probability\ that\ buyer\ i\ wins\ at\ least\ one\ auction}{probability\ that\ buyer\ i\ looses\ all\ auctions}.$$

Since all buyers are symmetric, we have $\lambda_i^{BLR}(v) = \lambda^{BLR}(v) \forall i \in \{1, 2, ..., n\}$, $v \in [\underline{v}, \overline{v}]$. Meanwhile, define

$$\lambda_i^{WLR}(v_i) \equiv \frac{probability\ that\ buyer\ i\ wins\ at\ least\ one\ auction}{probability\ that\ buyer\ i\ wins\ only\ one\ auction}.$$

as "the winning likelihood ratio"(WLR) for buyer i. Similarly, $\lambda_i^{WLR}(v) = \lambda^{WLR}(v)$.

4 Seller's Competition with Identical Private Values

4.1 Bidding by Multiauctioning

Assume the sellers have identical private values, which are independent of bidders' values. There are three key assumptions: (1) each seller knows all sellers' private values; (2) each buyer can multiauction[4] for maxizing his expected gain from the auctions; (3) all sellers auction their goods simultaneously.

The permission of multiauctioning implies that each buyer can participate in more than one auction simultaneously. In terms of feasibility, it is equivalent to sequentially participating in each auction if necessary for a buyer. In details, when the auction format is English (ascending), each buyer can bid in an auction at first, and once the current highest price is higher than his bid, he can submit the same bid in another auction at once until no current price is lower than his first bid in every auction. Then, he can submit a little higher bid and sequentially repeat it in each available auction again. Such process last until he wins a good or totally loses all auctions. Similarly, in Dutch format auctions, each buyer can take the parallel bidding process, except that once some buyer wins at a higher price which is bigger than his optimal bid in an auction, he will switch to another auction or equivalently, to the other auctions.

4.1.1 Buyers' Bidding Strategies and Sellers' Reserve Prices

According to Riley and Samuelson (1981), there is a common equilibrium bidding strategy in which each buyer makes a bid b_i $\forall i \in \{1, 2, ..., n\}$ when there are m auctions available, which is strictly increasing function of his value v_i:

[4] Similarly to the terminology of the literature of the two-sided market, we utilize the term "multiauctioning" for the buyers when they can participant in all available auctions simultaneously, which is parallel to multihoming..

$$b_i = b(v_i) \ \ \forall i \in \{1, 2, ..., n\}.$$

Since each seller knows all sellers' private values which are identical, then their optimal reserve prices are symmetric, that is, v_*. Thus, we have Proposition 1.

Proposition 1 *The common equilibrium bidding strategy for either high bid auction or English auction yields the expected revenue for each seller as*

$$\frac{n}{m}\int_{v_*}^{\overline{v}} [vf(v) + F(v) - 1]\left[1 - \left[1 - F^{n-1}(v)\right]^m\right] dv. \tag{1}$$

We put all proofs in the appendix. Proposition 1 is an extension of Riley and Samuelson (1981)'s proposition 1 in the setting of m sellers. Naturally, with permission of multiauctioning, each bidding buyer would like to submit a bid in an auction where the current highest bid lower than his previous bid in the other auctions. As a result, each seller's auction equivalently has n potential bidders. Hence, The expected payment of each buyer who can submit bids can be virtually viewed to be divided by all sellers on average, which may result in the diminishment of each seller's expected revenue.

Furthermore, with sellers' competition, each bidding buyer may possibly change his bidding strategy by shading his bid to a greater extent relative to that in one-seller case. Moreover, since multiauctioning can be viewed as sequentially participating in each auction, each buyer may bid for m times if he can't win once in the previous $m-1$ auctions. Hence, Propostion 2 supply such perception with an insightful result for considering a buyer's all possible bids in high bid auctions.

Proposition 2 *In high bid auction, the equilibrium bidding strategy of buyer i when there are j auctions available is*

$$b^j(v_i) = v_i - \frac{\int_{v_*}^{v_i}\left[1 - \left[1 - F^{n-m+j-1}(x)\right]^j\right] dx}{1 - \left[1 - F^{n-m+j-1}(v_i)\right]^j} \ \ \forall j \in \{1, 2, ..., m\}. \tag{2}$$

Moreover, his expected gain from auctions is strictly increasing in m if $\frac{\partial v_*}{\partial m} \leq 0$.

Intuitively, with more sellers, the probability that buyer wins becomes higher with more sellers, which results in a lower bid. So if a buyer bids more than once, his bidding strategy should be declining. However, such logic might not hold for some bidding buyers with quite low private values for their marginal increments of winning probability in an auction by enhancing their bids may be so high that they would like to bid more aggressively.

Obviously, the common bidding strategy in the Proposition reflects a quite different rationale from that in discriminatory multi-unit auctions in SIPV model where each bidder's estimating his winning probability involves an expected one by using the tool of order statistics (e.g. Milgrom and Weber, 2000). By contrast, in our model, each buyer substantially take the similar bidding strategy for each auction ecpect that his winning probability has changed when the number of available auctions decreases. In fact, such bidding rationale for each buyer is quite similar to the sequential first-price sealed-bid auctions in the literature of multi-unit auctions (Weber, 1983; Milgrom and Weber, 2000). However, each bidder when risk neutral use a symmetric equilibrium strategy in such auction which makes progressively higher bids in each successive stage of the auction, but the bidding strategy when sellers compete against each other may not be increasing for every buyer. This is one of the differences, induced by the strategic design of common reserve price by competitive sellers, between the auctionss with competitive sellers and the sequential multi-unit ones where the reserve price in each round or stage is set to be zero.

Let us focus our interest on the effects of sellers' competition on their optimal reserve price. Based on Proposition 1, we can easily derive such effects.

Proposition 3 *The optimal reserve price υ_* in either high bid auction or English auction, below which is not worthwhile bidding, is*

$$\upsilon_* = \frac{mF^{n-1}(\upsilon_*)}{1 - [1 - F^{n-1}(\upsilon_*)]^m}\upsilon_0 + \frac{1 - F(\upsilon_*)}{f(\upsilon_*)}. \tag{3}$$

Moreover, when Myerson's regular condition is satisfied, υ_* is decreasing in m if

$$\lambda_M^{BLR} \equiv \frac{1 - [1 - F^{n-1}(\upsilon_*)]^m}{[1 - F^{n-1}(\upsilon_*)]^m} \leq -\ln\left[1 - F^{n-1}(\upsilon_*)\right]^m, \tag{4}$$

while υ_* is increasing and each seller's total expected return is strictly decreasing in m if

$$\lambda_M^{BLR} \geq -\ln\left[1 - F^{n-1}(\upsilon_*)\right]^m. \tag{5}$$

In Proposition 3, λ_M^{BLR} is a buyer's BLR when all buyers can multihome the auctions. The proposition tells us that as the competition of sellers becomes more intense (m becomes larger), each seller's reserve price is decreasing in m if λ_M^{BLR} is relatively small, while υ_* is increasing and each seller's total expected return is strictly decreasing in m if λ_M^{BLR} relatively large. When λ_M^{BLR} is relatively small, the result is intuitive since driven by the more intense competition, each seller will lower his reserve price, which will also enhance each buyer's probability of winning at least one auction and subsequently, will increase his total expected return.

By contrast, when λ_M^{BLR} is relatively large, the probability of each buyer's winning at least one auction is increasing as m becomes larger, which induces that each seller's demand is less elastic though with indirectly more intense competition. As a result, the reserve price should be turned up for each seller. Such conclusion is quite interesting, even contradicts the traditional competition theory in Industrial Organization. However, from the perspective of two-sided market, such conclusion may be reasonable, for there are positive cross-group externalities from the sellers to buyers, and more sellers, more larger such positive externalities. Since each seller can only provide one unit good, and $n >> m$, all sellers don't have the incentive to lower their reserve prices to grab the others' buyer bases to earn more expected revenues when λ_M^{BLR} is relatively large. Meanwhile, in such a case, larger positive externalities induced by larger m make each seller's demand less elastic, which naturally leads to a higher reserve price.

Although the reserve price becomes higher when BLR is relatively large, the total expected return for each seller is decreasing. Because more intense competition among sellers induces larger positive externalities for each buyer, and as a result, directly affects each buyer's bidding strategy, and hence indirectly has a negative effect on each seller's total expected return, which coincides with the traditional competition theory.

To be emphasized, since we obtain such an equilibrium by a way equivalent to maximize the sum of all sellers' expected revenues and evenly allocate it to each seller, it can be viewed as the result of noncooperative collusion of all sellers.

4.1.2 Cross-Group Externalities in Online Auctions

In this section, we investigate the cross-group externalities between the sellers and the buyers in online auctions, which is so important that it can have effects on the equilibrium behaviors of both sides as well as the ones of the online auction platforms. Before discussing this problem, we should understand how changes of the numbers of both buyers and sellers affect each buyer's expected gain and each seller's total expected return.

Corollary 1 *When Myerson's regular condition is satisfied, υ_* is decreasing in n and strictly decreasing if $\upsilon_* < \overline{\upsilon}$. Moreover, each seller's total expected return is strictly increasing in n if*

$$\lambda^{WLR}(\upsilon_*) \equiv \frac{1 - [1 - F^{n-1}(\upsilon_*)]^m}{m\,[1 - F^{n-1}(\upsilon_*)]^{m-1}\,F^{n-1}(\upsilon_*)} \geq -\ln F^n(\upsilon_*). \tag{6}$$

On the one hand, the first result of Corollary 1 is quite counterintuitive for each seller's optimal reserve price would be increasing as the number of buyers increased intuitively. However, such a result is reasonable, because holding the reserve price constant, a seller's expected marginal benefit increases as n become larger by (3), so for maximizing his total expected return, he should lower his reserve price to make such marginal benefit decrease to the level equal to his private value which is his marginal cost. On the other hand, each seller's total expected return is strictly increasing in n is very intuitive for higher of the buyers' bidding as the number of buyers increases, which results in bigger of the probability that his good is auctioned out and higher of possible dealing price.

According to the literature of two-sided markets, the market of online auctions can be viewed as a two-sided market. However, such a market is special for the two sides "interact" by auctions. By the terminology of two-sided markets, we call the cross-group externalities of buyers on sellers "the bidder externalities", while the ones of sellers on buyers "the auctioneer externalities" if these externalities exist. The former reflects the effects of the change of the number of buyers on the total expected returns of sellers, while the latter reflects the effects of the change of the number of sellers on the expected gains of buyers. If the increase of the number of buyers improves each seller's total expected return, then we say that such bidder externalities are positive, while say that such bidder externalities are negative in the reverse case. Such sorting is similar to the auctioneer externalities. Thus, based on Proposition 2, 3 and Corollary 1, we summarize the results about these externalities in Theorem 1.

Theorem 1 *In either high bid auction or English auction, when Myerson's regular condition is satisfied, the auctioneer externalities are positive if* $\lambda_M^{BLR} \leq -\ln\left[1 - F^{n-1}(\upsilon_*)\right]^m$*, and the bidder externalities are positive if* $\lambda^{WLR}(\upsilon_*) \geq -\ln F^n(\upsilon_*)$*.*

Theorem 1 is very important for it permits us to use the analytical method of two-sided markets to investigate the online auctions, especially the behaviors of online auction platforms and corresponding effects on the behaviors of both sides. The results in the Theorem indicate that there do exist cross-group externalities in online auctions, and that each buyer can benefit from the auctioneer externalities if the bidding likelihood ratio for a typical buyer is not so large, and each seller can also benefit from the bidder externalities if the winning likelihood ratio is not so small.

To be emphasized, such cross-group externalities are endogenous other than the ones in the literature of two-sided markets which are exogenously supposed.[5] This result can be used to explain from a perspective of two-sided markets why by contrast to the traditional auctions, there are thousands of both buyers and sellers joining the online auction markets, and why they would like to join the big online auction sites or platforms, such as eBay, Yahoo, Ubid and Overstock, which coincides with Park (2001, 2002)'s empirical results. For instance, in 2008, there were about 90 millions of active users registering at eBay, and there were more than 10 millions categories of goods listed in eBay to be auctioned.

[5] In the literature of two-sided markets, e.g., Armstrong (2005), the utility of an agent in group 1, say, a buyer, is $u_1 \quad \alpha_1 n_2 - p_1$, the one in group 2, say, a seller, is $u_2 \quad \alpha_2 n_1 - p_2$, where α_i is the benefit that an agent in group i enjoys from each agent on the other side, n_j is the number of agents from group j, p_i is the platform's price to group i $\forall i \ / \ j \in \{ \ , \ \}$. In such model, $\alpha_i n_j$ captures the benefit of an agent in group i from group j's externalities, while $\alpha_j n_i$ the benefit from group i's externalities, which are exogenously described.

4.2 Bidding by Single-Auctioning

In this setting, each buyer can only participate in one seller's auction. For symmetric equilibrium, there must be a population of $\frac{n}{m}$ buyers who participate in each seller's auction. Thus, each seller sets his optimal reserve price as he was a monopolist. Therefore, we can directly obtain the corresponding conclusions as follows (since the results are direct, the corresponding proof is omitted).

Proposition 4 *In either high bid auction or English auction, the equilibrium yields the expected revenue for each seller as*

$$\frac{n}{m}\int_{\upsilon_*}^{\overline{\upsilon}} \left[\upsilon f\left(\upsilon\right)+F(\upsilon)-1\right] F^{n/m-1}(\upsilon)d\upsilon. \tag{7}$$

In high bid auction, the equilibrium bidding strategy of buyer i is

$$b(\upsilon_i)=\upsilon_i-\frac{\int_{\upsilon_*}^{\upsilon_i} F^{n/m-1}(x)dx}{F^{n/m-1}(\upsilon_i)}, \tag{8}$$

and $b(\upsilon_i)$ is strictly decreasing in m. Moreover, the optimal reserve price υ_* in either high bid auction or English auction is

$$\upsilon_*=\upsilon_0+\frac{1-F(\upsilon_*)}{f(\upsilon_*)}, \tag{9}$$

which is the same as the one with only one seller.

It seems surprisingly that the reserve price for each seller isn't affected by sellers' competition. For each buyer now can't gain more without participating in the other sellers' auctions. However, each buyer's bidding strategy is different from the one in one-seller case, and is strictly decreasing in the number of sellers. This is because in each seller's auction, the potential number of bidders decreases from n to $\frac{n}{m}$, that is, the competition among the buyers becomes less intense than the one in the case of bidding by multiauctioning, so each buyer bids to a less aggressive extent.

However, our interests focus on how the changes of the numbers of both buyers and sellers affect each buyer's expected gain and each seller's total expected return.

Proposition 5 *In the SIPV model without buyers' multiauctioning, each buyer's expected gain is strictly increasing in m in either high bid auction or English auction; while each seller's total expected return is strictly decreasing in m if $1+\ln F^{n/m}(\upsilon_*)\geq 0$, and is strictly increasing in n if $1+\ln F^{n/m}(\upsilon_*)>0$ and*

$$\frac{1-F(\upsilon_*)}{F(\upsilon_*)}\geq\frac{-\ln F(\upsilon_*)}{1+\ln F^{n/m}(\upsilon_*)}.$$

Proposition 5 is parallel with Proposition 2 and 3. When buyers can't multiauction, the auction situation for each seller is similar to that of one-seller case, except that there are much less buyers for each auction ($\frac{n}{m}$ other than n). Therefore, as m increases, each buyer's expected gain from participating in a specific auction improves for there are less competitors.

For sellers, if the reserve price is set low enough, namely, $1 + \ln F^{n/m}(\upsilon_*) \geq 0$, the competition effects will work and each seller's total expected return will strictly decrease as m increases. Moerover, if one-buyer bidding likelihood ratio $\frac{1-F(\upsilon_*)}{F(\upsilon_*)}$ is high enough (not less than $\frac{-\ln F(\upsilon_*)}{1+\ln F^{n/m}(\upsilon_*)}$), each seller's total expected return strictly increases in n, which implies that any seller's total expected return is correlated with not only the number of buyers, but the distribution of a typical buyer's private value.

Importantly, now the cross-group externalities arised by online auctioning exist as well. Based on Proposition 5, the auctioneer externalities are always positive, while the bidder externalities may be positve when one-buyer bidding likelihood ratio is high enough.

Theorem 2 *In the case of bidding by single-auctioning, in either high bid auction or English auction, the auctioneer externalities are positive, while the bidder externalities are positive if* $1 + \ln F^{n/m}(\upsilon_*) > 0$ *and* $\frac{1-F(\upsilon_*)}{F(\upsilon_*)} \geq \frac{-\ln F(\upsilon_*)}{1+\ln F^{n/m}(\upsilon_*)}$.

Such asymmetry of two-sided externalites is primarily the result of the substantial effects of the distributions of buyers' private values on each seller's reserve price design. Imagine, if there are too many buyers with lower private values, as n increases, the optimal reserve price of each seller is hardly adjusted or is adjusted downwards other than upwards, say, exponential distribution with a large parameter λ.

5 Sellers' Competition with Different Private Values

There is a more general case in online auctions that each seller only knows the distributions of the others' private values. Hence, in this section, we consider this more general case with two key assumptions: (1) each sellers' private value υ_0^j $\forall j \in \{1, 2, ..., m\}$ is an i.i.d. random variable for the others with a distribution function F_s and density f_s on interval $[\underline{\upsilon}, \overline{\upsilon}]$, (2) all sellers set their reserve prices at the same time, and once the reserve price is set by each seller, he can't be permitted to change it during the progress of his auction by the online auction platform. The latter assumption is not so restrictive for many online auction platforms do apply such auction format. According to Lucking-Reiley (2000), about 10 auction sites design the auction format in which the sellers should announce their reserve prices other than have secret reserve prices.

5.1 Bidding by Multiauctioning: Complete Information Case

First, we consider a preliminary case that each seller has different private value and they know the others' private values. For simplicity, assume after one of the sellers auction out his good, the others in progress still have the same distributions of buyers' private values. This assumption is quite helpful which makes the simultaneous auctioning equivalent to sequential auctioning from the seller with lowest reserve price to the one with highest reserve price. Since each seller may have different reserve price, without loss of generality, assume the seller j's private value is v_{0j}, and his optimal reserve price is increasing in v_{0j}. Then all reserve prices can be ordered as $v_*^m \le v_*^{m-1} \le ... \le v_*^1$. Obviously, the good with lower reserve price will auction out more easily and earlier. Hence, we assume that the good with lower reserve price will be auctioned out earlier than the one with higher reserve price. Thus, for the rest of buyers after the auction with v_*^{j+1} finished, they only face j sellers (see figure 1).

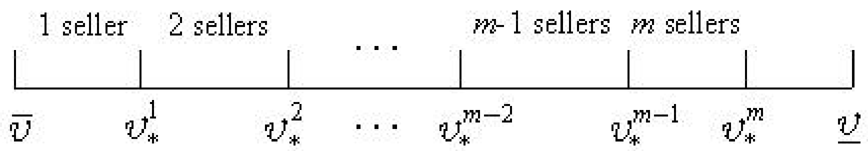

Figure 1: The potential sellers for the rest of buyers

Obviously and importantly, different buyers with different reservation values will have different expected payments. Therefore, we should segment a typical buyer into $m+1$ possible groups, see Figure 2.

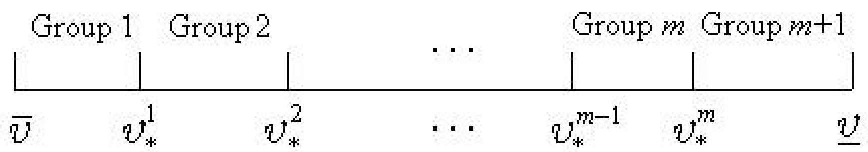

Figure 2: The segmentation of a typical buyer

In group 0, the buyer has private values lower than v_*^m thus wouldn't enter even an auction, while in group m, the buyer's private values is not less than v_*^1, which implies that he has the opportunity to win a good auctioned. Generally, define $v_*^0 \equiv \overline{v}$, then for buyer i with private value $v_*^j \le v_i < v_*^{j-1}$ $\forall j \in \{1, 2, ..., m\}$, he only enters $m-j+1$ auctions with reserve prices not higher than v_*^j. Hence, different seller may have different size of buyers participating in his auction.

5.1.1 Buyers' Bidding Strategies and Expected Gains

We still consider the auctions with the property that there is a common equilibrium bidding strategy which is strictly increasing function of each buyer's private value:

$$b_i^j = b_i^j(\upsilon_i) \ \ \forall i \in \{1, 2, ..., n\}, j \in \{1, 2, ..., m\}.$$

For seller j, assume that his ordered reserve price is υ_*^j. Since $m-j$ buyers might have obtained their desired goods, there might be $n-m+j$ buyers entering seller j's auction. Thus, for buyer i with υ_i which satisfies $\upsilon_*^j \leq \upsilon_i < \upsilon_*^{j-1}$, his probability to win a good is

$$1 - \Pi_{k=1}^{m-j+1}\left[1 - F^{n-k}(x)\right], \tag{10}$$

when he reports x given no goods have been auctioned out. He will sequentially enter from seller m's auction to seller j's until he wins a good. Then his total expected gain can be expressed as

$$\pi^m\left(x, \upsilon_i; \{\upsilon_*^h\}_{h=m}^j\right) = \upsilon_i\left[1 - \Pi_{k=1}^{m-j+1}\left[1 - F^{n-k}(x)\right]\right] - P_i^m\left(x; \{\upsilon_*^h\}_{h=m}^j\right), \tag{11}$$

where $P_i^m\left(x; \{\upsilon_*^h\}_{h=m}^j\right)$ is his expected payment. Accordingly, his first order condition is

$$\left.\frac{\partial \pi^m\left(x, \upsilon_i; \{\upsilon_*^h\}_{h=m}^j\right)}{\partial x}\right|_{x=\upsilon_i} = \upsilon_i \frac{d\left[1 - \Pi_{k=1}^{m-j+1}\left[1 - F^{n-k}(\upsilon_i)\right]\right]}{d\upsilon_i} - P_i^{m\prime}\left(\upsilon_i; \{\upsilon_*^h\}_{h=m}^j\right) = 0. \tag{12}$$

(12) must hold for the buyers belonging to group j whose private value satisfies $\upsilon_*^j \leq \upsilon_i < \upsilon_*^{j-1}$.

Furthermore, it is very important to obtain the ex ante expected payment boundary conditions for buyers with $\{\upsilon_*^j\}_{j=m}^1$ in turn. First, the boundary condition for seller m's auction can be expressed by

$$\upsilon_*^m F^{n-1}(\upsilon_*^m) - P^m(\upsilon_*^m; \upsilon_*^m) = 0. \tag{13}$$

Thus by (12) and (13), the expected payment of a typical buyer i with $\upsilon_i \in [\upsilon_*^m, \upsilon_*^{m-1})$ is

$$\begin{aligned} P_i^m(\upsilon_i; \upsilon_*^m) &= \upsilon_*^m F^{n-1}(\upsilon_*^m) + \int_{\upsilon_*^m}^{\upsilon_i} x dF^{n-1}(x) \\ &= \upsilon_i F^{n-1}(\upsilon_i) - \int_{\upsilon_*^m}^{\upsilon_i} F^{n-1}(x)dx. \end{aligned} \tag{14}$$

Similarly, the expected payment of a typical buyer i with $\upsilon_i \in [\upsilon_*^j, \upsilon_*^{j-1})$ is

$$P_i^m\left(\upsilon_i; \{\upsilon_*^h\}_{h=m}^j\right) = P^m\left(\upsilon_*^j; \{\upsilon_*^h\}_{h=m}^j\right) + \int_{\upsilon_*^j}^{\upsilon_i} x d\left[1 - \Pi_{k=1}^{m-j+1}\left[1 - F^{n-k}(x)\right]\right]. \tag{15}$$

Given $m-j$ goods have been auctioned out, the expected gain of a typical buyer i with $\upsilon_i \in [\upsilon_*^j, \upsilon_*^{j-1})$ from the rest auctions is

$$\pi^j\left(x, \upsilon_i; \left\{\upsilon_*^h\right\}_{h=m}^j\right) = \upsilon_i F^{n-m+j-1}(x) - P_i^j\left(x; \left\{\upsilon_*^h\right\}_{h=m}^j\right). \tag{16}$$

The corresponding boundary condition for seller j's auction can be expressed by

$$\upsilon_*^j F^{n-m+j-1}(\upsilon_*^j) - P^j\left(\upsilon_*^j; \left\{\upsilon_*^h\right\}_{h=m}^j\right) = 0. \tag{17}$$

Combining (16) and (17), by FOC condition of $\pi^j\left(x, \upsilon_i; \left\{\upsilon_*^h\right\}_{h=m}^j\right)$ we have

$$P_i^j\left(\upsilon_i; \left\{\upsilon_*^h\right\}_{h=m}^j\right) = \upsilon_i F^{n-m+j-1}(\upsilon_i) - \int_{\upsilon_*^j}^{\upsilon_i} F^{n-m+j-1}(x)dx. \tag{18}$$

Since the buyer with υ_*^j sequentially enters from seller m's auction to seller j's, we have

$$P^m\left(\upsilon_*^j; \left\{\upsilon_*^h\right\}_{h=m}^j\right) = P^m\left(\upsilon_*^j; \upsilon_*^m\right) + \sum_{l=1}^{m-j}\left[\Pi_{k=1}^l\left[1 - F^{n-k}(\upsilon_*^j)\right]\right] P^{m-l}\left(\upsilon_*^j; \left\{\upsilon_*^h\right\}_{h=m}^{m-l}\right).$$

Define $P^m\left(\upsilon_*^j; \upsilon_*^m\right) \equiv \left[\Pi_{k=1}^{l=0}\left[1 - F^{n-k}(\upsilon_*^j)\right]\right] P^{m-l}\left(\upsilon_*^j; \left\{\upsilon_*^h\right\}_{h=m}^{m-l}\right)$, then

$$P^m\left(\upsilon_*^j; \left\{\upsilon_*^h\right\}_{h=m}^j\right) = \sum_{l=0}^{m-j}\left[\Pi_{k=1}^l\left[1 - F^{n-k}(\upsilon_*^j)\right]\right] P^{m-l}\left(\upsilon_*^j; \left\{\upsilon_*^h\right\}_{h=m}^{m-l}\right). \tag{19}$$

Hence, by (14), (15) and (18), we obtain

$$\begin{aligned}
P_i^m\left(\upsilon_i; \left\{\upsilon_*^h\right\}_{h=m}^j\right) &= \sum_{l=0}^{m-j}\left[\Pi_{k=1}^l\left[1 - F^{n-k}(\upsilon_*^j)\right]\right] P^{m-l}\left(\upsilon_*^j; \left\{\upsilon_*^h\right\}_{h=m}^{m-l}\right) \\
&\quad + \int_{\upsilon_*^j}^{\upsilon_i} xd\left[1 - \Pi_{k=1}^{m-j+1}\left[1 - F^{n-k}(x)\right]\right] \\
&= \sum_{l=0}^{m-j}\left[\Pi_{k=1}^l\left[1 - F^{n-k}(\upsilon_*^j)\right]\right]\left[\upsilon_*^j F^{n-l-1}(\upsilon_*^j) - \int_{\upsilon_*^{m-l}}^{\upsilon_*^j} F^{n-l-1}(x)dx\right] \\
&\quad + \int_{\upsilon_*^j}^{\upsilon_i} xd\left[1 - \Pi_{k=1}^{m-j+1}\left[1 - F^{n-k}(x)\right]\right].
\end{aligned} \tag{20}$$

Therefore, by sequential entering strategy, each buyer's total expected gain can be derived if all sellers set different optimal reserve prices.

Proposition 6 *If all sellers set different reserve prices, then for buyer i with $\upsilon_i \in [\upsilon_*^j, \upsilon_*^{j-1})$ $\forall j \in \{1, 2, ..., m\}$, his total expected gain is*

$$\begin{aligned}\pi^m\left(\upsilon_i, \upsilon_i; \{\upsilon_*^h\}_{h=m}^j\right) &= \upsilon_*^j\left[1-\Pi_{k=1}^{m-j+1}\left[1-F^{n-k}(\upsilon_*^j)\right]\right] + \int_{\upsilon_*^j}^{\upsilon_i}\left[1-\Pi_{k=1}^{m-j+1}\left[1-F^{n-k}(x)\right]\right]dx \\ &\quad - \sum_{l=0}^{m-j}\left[\Pi_{k=1}^{l}\left[1-F^{n-k}(\upsilon_*^j)\right]\right]\left[\upsilon_*^j F^{n-l-1}(\upsilon_*^j) - \int_{\upsilon_*^{m-l}}^{\upsilon_*^j} F^{n-l-1}(x)dx\right] \quad (21)\end{aligned}$$

Intuitively, the total expected gain of buyer i with $\upsilon_i \in [\upsilon_*^j, \upsilon_*^{j-1})$ is affected by $m - j + 1$ reserve prices not higher than υ_i. In the case of sellers with identical private values, when there are j sellers are available and $n - m + j$ buyers left, the expected payment of a buyer i, whose private value is not lower than the reserve price, is $\upsilon_i\left[1-\left[1-F^{n-m+j-1}(\upsilon_i)\right]^j\right] - \int_{\upsilon_*}^{\upsilon_i}\left[1-\left[1-F^{n-m+j-1}(x)\right]^j\right]dx$ $\forall j \in \{1, 2, 3, 4, 5\}$ for there is a common reserve price for all sellers. However, in the current case, the expected payment of each left buyer, who can bid for at least one still available auction, is correlated with the reserve price(s) set in the previous acomplished auction(s) since the sellers have different private values, though except for the one of each bidding seller in the auction with lowest reserve price. Such correlation between each buyer's expected payment and the previous reserve price(s) makes each buyer take quite different bidding strategy rather than that in the case of sellers with identical private values. Furthermore, It also makes each buyer's bidding strategy differ from that in the sequential first-price mult-unit auctions where there are either no reserve price at all or a common one for only one seller.

Now, let us investigate each buyer's equilibrium bidding strategy in high bid auction. Based on (20), when there are $m - a$ goods left $\forall a \in \{0, 1, ..., m - j\}$, the expected payment of a typical buyer i with $\upsilon_i \in [\upsilon_*^j, \upsilon_*^{j-1})$ can be expressed by

$$\begin{aligned}P_i^{m-a}\left(\upsilon_i; \{\upsilon_*^h\}_{h=m}^j\right) &= \sum_{l=a}^{m-j}\left[\Pi_{k=a+1}^{l}\left[1-F^{n-k}(\upsilon_*^j)\right]\right]\left[\upsilon_*^j F^{n-l-1}(\upsilon_*^j) - \int_{\upsilon_*^{m-l}}^{\upsilon_*^j} F^{n-l-1}(x)dx\right] \\ &\quad + \int_{\upsilon_*^j}^{\upsilon_i} x d\left[1-\Pi_{k=a+1}^{m-j+1}\left[1-F^{n-k}(x)\right]\right] \quad (22)\end{aligned}$$

Since the buyer pays if and only if he is the high bidder, his expected payment is

$$P_i^{m-a}\left(\upsilon_i; \{\upsilon_*^h\}_{h=m}^j\right) = \left[1-\Pi_{k=a+1}^{m-j+1}\left[1-F^{n-k}(\upsilon_i)\right]\right] b_i^{m-a}(\upsilon_i). \quad (23)$$

Therefore, combining (22) and (23), we obtain Proposition 7 as each buyer's sequential equilibrium bidding strategy.

Proposition 7 $\forall j \in \{1, 2, ..., m\}$, $a \in \{0, 1, ..., m - j\}$, *a typical buyer i with $\upsilon_i \in [\upsilon_*^j, \upsilon_*^{j-1})$ takes a sequential equilibrium bidding strategy as following: if there are*

$m-a$ goods left, his bidding is

$$b_i^{m-a}(\upsilon_i) = \upsilon_i - \frac{\int_{\upsilon_*^j}^{\upsilon_i}\left[1-\Pi_{k=a+1}^{m-j+1}\left[1-F^{n-k}(x)\right]\right]dx}{1-\Pi_{k=a+1}^{m-j+1}\left[1-F^{n-k}(\upsilon_i)\right]} - \frac{\upsilon_*^j\left[1-\Pi_{k=a+1}^{m-j+1}\left[1-F^{n-k}(\upsilon_*^j)\right]\right]}{1-\Pi_{k=a+1}^{m-j+1}\left[1-F^{n-k}(\upsilon_i)\right]}$$
$$+\frac{\sum_{l=a}^{m-j}\left[\Pi_{k=a+1}^{l}\left[1-F^{n-k}(\upsilon_*^j)\right]\right]\left[\upsilon_*^j F^{n-l-1}(\upsilon_*^j)-\int_{\upsilon_*^{m-l}}^{\upsilon_*^j}F^{n-l-1}(x)dx\right]}{1-\Pi_{k=a+1}^{m-j+1}\left[1-F^{n-k}(\upsilon_i)\right]}. \quad (24)$$

Moreover, the monotonicity of $b_i^{m-a}(\upsilon_i)$ in $m-a$ may depend on υ_i, n, m and $\left\{\upsilon_*^h\right\}_{h=m}^1$.

Surprisingly, Proposition 7 establishes that each buyer's sequential bidding strategy may possibly not be ascending as the number of goods not auctioned out decreases, which is quite counterintuitive. Intuitively, less goods left, more difficult to win an auction, and the intensity of corresponding competition among buyers almost unchanges for $n >> m$. As a result, each buyer who enters such an auction left will bid more aggressively in order to maximize his total expected gain. Such logic seems so reasonableLet that we can easily understand this point by demonstratiing an example of high bid auction. There are two sellers, say A and B, A's reserve price is 50 dollars, while B's is 80 dollars. For a buyer whose private value is 80 dollars, he will firstly bid for the good with reserve price of 50 dollars, and if he lost, he would bid for seller B's good. Since his private value is equal to seller B's reserve price, then he can only bid 80 dollars for seller B's good given he lost seller A's auction, while obviously, from our proposition, we know he will bid less than 80 dollars for seller A's good. Thus his sequential bidding strategy is ascending.

However, such logic ignores the impacts of both υ_i and $\left\{\upsilon_*^h\right\}_{h=m}^1$, which may be quite significant to decide how the bidding strategy is for the corresponding buyer. To consider both of them, imagine a buyer with private value just slightly higher than υ_*^{m-1} when m is not too small and υ_*^m is slightly lower than υ_*^{m-1}. On the one hand, given he has lost the first auction of the seller with υ_*^m, in order to optimize his expected gain from the second and his last auction, he should increase his bid. Nevertheless, since the difference between υ_*^{m-1} and υ_*^m is small enough, so the buyer may bid $b_i^m(\upsilon_i)$ which is slightly higher than υ_*^m. Obviously, when he faces his last auction, his probability to win the auction decreases to a great extent, hence he should bid by a big jump relatve to $b_i^m(\upsilon_i)$. According to our supposition, his second (last) bid must exceed his private value, so he only can bid υ_i, which is not the optimal. On the other hand, the marginal probability to win a good increases by enhancing what he bids in his first auction to a larger degree than the one induced by the identical increment of what he bids in his second auction, so he should ajdust his first-auction bid upwards and second-auction bid downwards. Such adjustment may last until the marginal winning probability of each auction equals to each other, and result in that the first optimal bid is higher than the second since the identical bid in his first auction may generate a much higher marginal winning probability than the one in his second auction. Naturally, if the difference between υ_*^{m-1} and υ_*^m is big enough, such decending bidding strategy may not be applied by the buyer. While for a certain buyer with much high private value which is close to $\overline{\upsilon}$ and higher than υ_*^1, the failure in the first auction has a relatively small effect on his later winning probability or even a slight effect when m is large enough, so as a whole, ascending bidding strategy may be optimal for him. Meanwhile, applying such analytical logic, the buyers with intermediate private values may bid ascendingly, descendingly, or nonmonotonely.

Based on the above analysis, we cann't explicitly obtain a monotone bidding strategy in such sequential equilibrium. For the complexity of $b_i^{m-a}(\upsilon_i)$ in Proposition 7, we just propose the reasonable conjecture below to gain an insight into each buyer's bidding strategy.

Conjecture 1 *There may be two critical private values of buyers in high bid auctions:* $\underline{\upsilon}_M = \underline{V}(n, m, \{\upsilon_*^h\}_{h=m}^1)$ *and* $\overline{\upsilon}_M = \overline{V}(n, m, \{\upsilon_*^h\}_{h=m}^1)$, $\upsilon_*^m \leq \underline{\upsilon}_M < \overline{\upsilon}_M \leq \overline{\upsilon}$, *such that each buyer with* $\upsilon_i \leq \underline{\upsilon}_M$ *takes the descending bidding strategy from the auction with lowest reserve price to the one with highest reserve price, and each buyer with* $\upsilon_i \geq \overline{\upsilon}_M$ *takes the ascending bidding strategy, while the bidding strategy of each buyer with* $\underline{\upsilon}_M < \upsilon_i < \overline{\upsilon}_M$ *is ambiguous and can be only decided by Proposition 7.*

According to our above analysis, if there does be $\underline{\upsilon}_M$ and $\overline{\upsilon}_M$, $\overline{\upsilon}_M$ may be decreasing in m, while $\underline{\upsilon}_M$ may be increasing in n since the competition among buyers become more intense as n turns to be bigger. Meanwhile, both $\underline{\upsilon}_M$ and $\overline{\upsilon}_M$ may be decreasing as the difference between each pair of adjacent reserve prices becomes larger, which means more buyers apply the ascending bidding strategy and less buyers apply the converse one.

5.1.2 Sellers' Reserve Prices and Expected Revenues

Since the auction with lower reserve price may end earlier than the one with higher reserve price, then sellers with different private values may possibly have different expected revenues. For seller j with υ_*^j, $n - m + j$ potential buyers left will enter his auction and each will take the bidding strategy $b_i^j(\upsilon_i)$ $\forall i \in \{1, 2, ..., n - m + j\}$ if not all the reserve prices are identical, so his expected revenue from buyer i through auction is

$$R^j\left(\upsilon_i; \{\upsilon_*^h\}_{h=m}^j\right) = \int_{\upsilon_*^j}^{\overline{\upsilon}} \left[\Pi_{k=1}^{m-j}\left[1 - F^{n-k}(\upsilon_i)\right]\right] P_i^j\left(\upsilon_i; \{\upsilon_*^h\}_{h=m}^j\right) f(\upsilon_i)\, d\upsilon_i.$$

Hence, by (18), we obtain all sellers' expected revenues from their own auctions as following.

Proposition 8 *When all buyers take their common sequential equilibrium bidding strategies, in either the high bid auctions or English auctions,* $\forall j \in \{1, 2, ..., m\}$, *seller j with υ_*^j obtains his expected revenue of*

$$R^j\left(\left\{\upsilon_*^h\right\}_{h=m}^j\right)=(n-m+j)\int_{\upsilon_*^j}^{\overline{\upsilon}}\left[\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon)\right]\right]\left[\begin{array}{c}\upsilon F^{n-m+j-1}(\upsilon)-\\ \int_{\upsilon_*^j}^{\upsilon}F^{n-m+j-1}(x)dx\end{array}\right]f\left(\upsilon\right)d\upsilon \tag{25}$$

if all the reserve prices are different.

The Proposition implies that although seller j expected revenue from his auction is only explicitly affected by his own reserve price it does be implicitly affected by the reserve prices of the other sellers through its' order in all reserve prices. Such impacts reflect the effects of sellers' competition, which is even unnecessary to be considered in sequential multi-unit auctions with one seller.

Assume all sellers' private values can be ordered like $\upsilon_0^m\leq\upsilon_0^{m-1}\leq...\leq\upsilon_0^1$. then $\upsilon_{0j}=\upsilon_0^j$. Now interestingly, the probability that seller j will still retain his good after auction is more complicated than one seller case, even than m sellers with identical private values, which can be expressed by

$$P^j\left(\upsilon_0^j,\left\{\upsilon_*^h\right\}_{h=m}^j\right)=1-\Pi_{k=0}^{m-j}\left[1-F^{n-k}(\upsilon_*^{m-k})\right]. \tag{26}$$

Thus, based on (25) and (26), seller j's total expected return is

$$\begin{aligned}R^j\left(\upsilon_0^j;\left\{\upsilon_*^h\right\}_{h=m}^j\right) &= \upsilon_0^j\left[1-\Pi_{k=0}^{m-j}\left[1-F^{n-k}(\upsilon_*^{m-k})\right]\right]+\\ &\quad (n-m+j)\int_{\upsilon_*^j}^{\overline{\upsilon}}\left[\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon)\right]\right]\left[\begin{array}{c}\upsilon F^{n-m+j-1}(\upsilon)-\\ \int_{\upsilon_*^j}^{\upsilon}F^{n-m+j-1}(x)dx\end{array}\right]f\left(\upsilon\right)d\upsilon.\end{aligned} \tag{27}$$

We therefore have the following result about each seller's optimal reserve price.

Proposition 9 *Given all optimal reserve prices are different, the seller j's optimal reserve price υ_*^j $\forall j\in\{1,2,...,m\}$ in either high bid auction or English auction satisfies*

$$\upsilon_*^j=\upsilon_0^j\frac{\Pi_{k=0}^{m-j-1}\left[1-F^{n-k}(\upsilon_*^{m-k})\right]}{\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon_*^j)\right]}+\frac{\int_{\upsilon_*^j}^{\overline{\upsilon}}\left[\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon)\right]\right]f(\upsilon)d\upsilon}{\left[\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon_*^j)\right]\right]f(\upsilon_*^j)}, \tag{28}$$

which is dependent of the number of all buyers and the other $m-j$ lower reserve prices.

Proposition 9 indicates that when the sellers' private values are different, except seller m, the other sellers' optimal reserve prices are correlated with the sellers whose reserve prices are lower, which is distinguished from the case of sellers' identical private values. Since a typical seller j may auction out his good only after the other $m-j$ goods with lower reserve prices have been auctioned out, his optimal reserve price is an expected one. Once the left buyers enter his auction, seller j has the market power equivalent to a monopolist, so he can set his reserve price by the pricing rule of monopoly. Based on (28), seller j's expected marginal cost $\overline{MC}^j$ is

$$\overline{MC}^j = \upsilon_0^j \Pi_{k=0}^{m-j-1} \left[1 - F^{n-k}(\upsilon_*^{m-k})\right],$$

and his expected marginal benefit $\overline{MR}^j$ is

$$\overline{MR}^j = \upsilon_*^j \Pi_{k=1}^{m-j} \left[1 - F^{n-k}(\upsilon_*^j)\right] - \frac{1}{f(\upsilon_*^j)} \int_{\upsilon_*^j}^{\overline{\upsilon}} \left[\Pi_{k=1}^{m-j} \left[1 - F^{n-k}(\upsilon)\right]\right] f(\upsilon) d\upsilon.$$

Thus, his optimal setting is to make $\overline{MR}^j = \overline{MC}^j$ hold, which is the economic sense of (28).

Such "monopoly power" makes the typical seller j can announce a reserve price strictly greater than his personal valuation, just like the cases of traditional auctions and online auctions with seller's identical private values for $\Pi_{k=0}^{m-j-1} \left[1 - F^{n-k}(\upsilon_*^{m-k})\right] > \Pi_{k=1}^{m-j} \left[1 - F^{n-k}(\upsilon_*^j)\right]$. In the case of tradition auctions, say, Riley and Samuelson (1981), $\frac{1-F(\upsilon_*^j)}{f(\upsilon_*^j)}$ is the accumulative information rent of the buyers whose private values are higher than υ_*^j. However, since $\Pi_{k=1}^{m-j} \left[1 - F^{n-k}(\upsilon)\right]$ decreases in υ, the second part of RHS in (28) is less than $\frac{1-F(\upsilon_*^j)}{f(\upsilon_*^j)}$. This implies that conditional information rent seller j can extract decreases as seller j's reserve price is ranked higher. By contrast, he puts bigger weight on his private value to optimize his reserve price.

However, some important problems still confront us: Whether setting different reserve prices is an equilibrium? If it does be an equilibrium, is it the only one? Based on the analysis of the case that all sellers' private values are identical, setting a common reserve price may intuitively be a reasonable equilibrium, which is a tacit collusion result of all sellers. If such a result exists, the optimal common reserve price should be not less than the most highest one of all sellers' private values. Our intuition can be verified by Proposition 10.

Proposition 10 *If $\upsilon_{01} > \upsilon_{02} > ... > \upsilon_{0m}$, then in either high bid auction or English auction, $\{\upsilon_*^j\}_{j=m}^1$ may be an equilibrium. Meanwhile, if $\upsilon_*^M \geq \upsilon_{01}$, then all sellers set a common reserve price as*

$$\upsilon_*^M = \left(\sum\nolimits_{j=m}^1 \upsilon_{0j}\right) \frac{F^{n-1}(\upsilon_*^M)}{1 - \left[1 - F^{n-1}(\upsilon_*^M)\right]^m} + \frac{1 - F(\upsilon_*^M)}{f(\upsilon_*^M)} \tag{29}$$

and allocate the real jth highest bid to the seller with υ_{0j} $\forall j \in \{1, 2, ..., m\}$ if feasible may also be an equilibrium that is a result of tacit collusion of all sellers, under which υ_*^M is strictly decreasing in m. Furthermore, if υ_0 is a typical seller's private value and υ_* is his optimal reserve price in the parallel case that all sellers have identical private values, when $\sum_{j=m}^1 \upsilon_{0j}/m \geq \upsilon_0$, $\upsilon_*^M \geq \upsilon_*$, and vice versa.

The equilibria in Proposition 10 consist of two extremes: the former is the one that all sellers wouldn't like to collude with each other, and the latter is the one that all sellers tacitly collude. Naturally, when m is quite large, the collusion is very hard to achieve, which is the case that can be described by the former; while when m is small enough, all sellers can easily achieve collusion, which corresponds to the latter.

5.2 Bidding by Multiauctioning: Incomplete Information Case

It's time for us to turn to the asymmetric information case. The difference between complete information case and incomplete information one only lies in the behaviors of sellers, because current situation is indifferent for all buyers who have known all the reserve prices before they begin to bid. Moreover, since each seller doesn't exactly know each other, a tacit collusion equilibrium may be hardly achieved, so we ignore corresponding analysis.

5.2.1 Sellers' Expected Revenues

Based on the results in the preliminary case, for seller l with private value υ_{0l} $\forall l \in \{1, 2, ..., m\}$, the order of his reserve price in all sellers' have m possible cases. Namely, $\upsilon_{0l} = \upsilon_0^m$, or $\upsilon_{0l} = \upsilon_0^{m-1}$,. . . , or $\upsilon_{0l} = \upsilon_0^1$, so is the order of his reserve price. Assume his corresponding reserve price is υ_*^l, then the probability of $\upsilon_*^l = \upsilon_*^j$ is

$$P_s\left(\upsilon_*^l = \upsilon_*^j\right) = C_{m-1}^{j-1}\left[1 - F_s(\upsilon_*^l)\right]^{j-1} F_s^{m-j}(\upsilon_*^l)\ \forall j \in \{1, 2, ..., m\}. \qquad (30)$$

Accordingly, his ex ante conditional expected revenue $R^j\left(\left\{\upsilon_*^h\right\}_{h=m}^j \left|\upsilon_*^l = \upsilon_*^j\right.\right)$ from his auction is determined by (25). Thus his ex ante expected revenue by auctioning is

$$E_{\upsilon_*^{-l}} R^l(\upsilon_*^l, \upsilon_*^{-l}) = \sum_{j=m}^{1}\left[C_{m-1}^{j-1}\left[1 - F_s(\upsilon_*^l)\right]^{j-1} F_s^{m-j}(\upsilon_*^l)\right] R^j\left(\left\{\upsilon_*^h\right\}_{h=m}^j \left|\upsilon_*^l = \upsilon_*^j\right.\right).$$

Therefore, according to the results of Proposition 8, we directly derive each seller's ex ante expected revenue by auctioning.

Proposition 11 *When each seller has asymmetric information about the other sellers' private values and all buyers take their common sequential equilibrium bidding strategies, in either the high bid auctions or English auctions,* $\forall l \in \{1, 2, ..., m\}$*, seller* l *with* υ_*^l *obtains his ex ante expected revenue of*

$$E_{\upsilon_*^{-l}} R^l(\upsilon_*^l, \upsilon_*^{-l}) = \sum_{j=m}^{1}\left[\begin{array}{c}\left[C_{m-1}^{j-1}\left[1 - F_s(\upsilon_*^l)\right]^{j-1} F_s^{m-j}(\upsilon_*^l)\right](n-m+j) \\ \int_{\upsilon_*^l}^{\overline{\upsilon}}\left[\Pi_{k=1}^{m-j}\left[1 - F^{n-k}(\upsilon)\right]\right]\left[\begin{array}{c}\upsilon F^{n-m+j-1}(\upsilon) - \\ \int_{\upsilon_*^l}^{\upsilon} F^{n-m+j-1}(x)dx\end{array}\right] f(\upsilon)\, d\upsilon\end{array}\right]. \qquad (31)$$

Proposition 11 implies that seller l's ex ante expected revenue from his auction explicitly depends on his own private value (or reserve price) and the distributions of other sellers' private values (or reserve prices). In essence, each seller's ex ante expected revenue is the weighed summation of the expected revenues in m possible cases in which his reserve price is ranked from the highest to the lowest, respectively. So for a seller with the lowest private value. or even with relative low one, in the corresponding complete information case, he is inclined to estimate a relatvely higher expected revenue, while a relatively lower one for a seller with the highest private value, or even the relative high one. These ex ante extimations obviously result in the shrinking of the difference between both, the ex ante expected revenues of the sellers with the private values in two ends and the difference between each pair of sellers with adjacent reserve prices on average as well.

5.2.2 Sellers' Reserve Prices

In such an asymmetric information case, each seller sets his reserve price to maximize his ex ante total expected return, so his setting is just ex ante optimal. If we require his equilibrium reserve price is ex post optimal, we have to permit him to have the right to change his reserve price when he knows the other sellers' reserve prices at the beginning of their auctions. Such a requirement will make our analysis quite complicated, hence we only focus on the ex ante equilibrium. Of course, at most cases, such an ex ante equilibrium for each seller is suboptimal.

For convenience, we define

$$P(\upsilon_*^l; \upsilon_*^l = \upsilon_*^j) \equiv \left[\Pi_{k=1}^{m-j} \left[1 - F^{n-k}(\upsilon_*^l)\right]\right] F^{n-m+j-1}(\upsilon_*^l) \tag{32}$$

as the probability that the buyer with private value υ_*^l wins the auction of seller l whose reserve price satisfies $\upsilon_*^l = \upsilon_*^j$ conditional on his loss of previous auctions with reserve prices lower than υ_*^l, and define

$$P^j(\upsilon_*^l; \{\upsilon_*^h\}_{h=m}^j) \equiv \left[\Pi_{k=0}^{m-j-1} \left[1 - F^{n-k}(\upsilon_*^{m-k})\right]\right] F^{n-m+j-1}(\upsilon_*^l) \tag{33}$$

as the probability that the buyer with private value $\upsilon_*^l = \upsilon_*^j$ wins seller l's auction with reserve price υ_*^l conditional on that the goods with reserve prices lower than υ_*^l have been auctioned out. Meanwhile, when $\upsilon_*^l = \upsilon_*^j$, we rewrite $R^j\left(\{\upsilon_*^h\}_{h=m}^j\right)$ as $R^j\left(\{\upsilon_*^h\}_{h=m}^j \left|\upsilon_*^l = \upsilon_*^j\right.\right)$. Thus, based on (27), a typical seller l's optimal reserve price can be designed as follow.

Proposition 12 $\forall l \in \{1, 2, ..., m\}$, *the optimal reserve price of seller with* υ_{0l} *in* *either high bid auction or English auction satisfies*

$$\upsilon_*^l = \upsilon_{0l}\frac{\left[\begin{array}{c}\frac{f_s(\upsilon_*^l)}{f(\upsilon_*^l)}\sum_{j=m}^{1}\left[\frac{m-j}{F_s(\upsilon_*^l)}-\frac{j-1}{1-F_s(\upsilon_*^l)}\right]P_s\left(\upsilon_*^l=\upsilon_*^j\right)P^j(\upsilon_{0l},\left\{\upsilon_*^h\right\}_{h=m}^j)\\ +\sum_{j=m}^{1}(n-m+j)P_s\left(\upsilon_*^l=\upsilon_*^j\right)P^j(\upsilon_*^l;\left\{\upsilon_*^h\right\}_{h=m}^j)\end{array}\right]}{\sum_{j=m}^{1}(n-m+j)P_s\left(\upsilon_*^l=\upsilon_*^j\right)P(\upsilon_*^l;\upsilon_*^l=\upsilon_*^j)}+$$
$$\frac{\sum_{j=m}^{1}P_s\left(\upsilon_*^l=\upsilon_*^j\right)\left[\begin{array}{c}f_s(\upsilon_*^l)\left[\frac{m-j}{F_s(\upsilon_*^l)}-\frac{j-1}{1-F_s(\upsilon_*^l)}\right]R^j\left(\left\{\upsilon_*^h\right\}_{h=m}^j\middle|\upsilon_*^l=\upsilon_*^j\right)\\ +(n-m+j)F^{n-m+j-1}(\upsilon_*^l)\int_{\upsilon_*^l}^{\overline{\upsilon}}\left[\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon)\right]\right]f(\upsilon)\,d\upsilon\end{array}\right]}{f(\upsilon_*^l)\sum_{j=m}^{1}(n-m+j)P_s\left(\upsilon_*^l=\upsilon_*^j\right)P(\upsilon_*^l;\upsilon_*^l=\upsilon_*^j)}. \tag{34}$$

Moreover, it depends on his own private value, the distributions,of the other sellers' private values, and the numbers of both buyers and sellers.

Obviously, by (34), when all sellers have identical private values, they will design the same reserve price, otherwise, at least some of them may set different reserve prices. Furthermore, υ_*^l is increasing in υ_{0l}, which is quite intuivtive. Based on the above analysis about the relationships of their ex ante expected revenues against the ones in corresponding complete information case, it is reasonable to predict that the differences of reserve prices between the sellers with the highest and lowest private values will shrink, as well as the average difference between each pair of adjacent reserve prices. That is, the lowest reserve price will ascend, the highest will descend, and all reserve prices become closer to each other on average. Such changes may lead to the improvement of the total expected gain of each buyer in relative high end, but deterioration of each buyer in relative low end whose prive value is not lower than the lowest reserve price in the complete information case, since each buyer will bid more aggressively for the first auction with lowest reserve price but less aggressively for the last one with highest reserve price. However, the ex ante total expected return of each seller relative to that in the complete information case is ambiguous yet.

6 Simulation of Complete Information Cases

For better understanding the results of competitive sellers with different private values in online auctions, we use a specific numerical simulation to demonstrate each buyer's bidding strategy, each seller's design of reserve price, and the auctioneer and bidder externalities as the number of corresponding side changes.

Consider a Kingston DataTraveler 101 U-disc with 4GB online auction[6]. Assume that a typical buyer's private value has a uniform distribution on $[\underline{\upsilon}, \overline{\upsilon}] = [0, 100]$ RMBs. For convenience of comparision, we treat the one-seller case as the benchmark

[6] The online fixed price of a Kingston DataTraveler 101 U-disc with 4GB is about in the interval of [illegible], [illegible] RMBs in China On August, 2009.

where there are $n = 100$ buyers. The online auction platform is set to apply high bid (Dutch) auction format for demonstrating each buyer's bidding strategy.[7] In the numerical simulation, we compare four cases of one seller, competitive sellers with buyers' multiauctioning, heterogeneous reserve prices as well as common collusive one with complete information. In the last three cases, we set $m = 5$, and use Genetic Algorithm to find out the approximate solution of each seller's optimal reserve price for the number of potential solutions are too large. Furthermore, we demonstrate the cross-group externalities on both sides in the buyers' multiauctioning case.

6.1 Comparison of Cases with Common Reserve Prices

For convenience of comparing the cases of one seller (case I), competitive sellers with buyers' multiauctioning (case II) and common collusive reserve price with complete information (case III), we set $\upsilon_0 = 40$ RMBs in the first two cases, and set $\{\upsilon_0^5, \upsilon_0^4, \upsilon_0^3, \upsilon_0^2, \upsilon_0^1\} = \{40, 45, 50, 55, 60\}$ RMBs in the collusion case. Meanwhile, set buyers' private values as $\upsilon_i = 100/n \times i \ \forall i \in \{1, 2, ..., 100\}$.

According to Proposition 1-3, Proposition 10 and corresponding proofs, we can compute and compare each buyer's bids, expected gain, each seller's reserve price and total expected gain in the three cases (see table 2). To avoid the redundant bidding schedule of each buyer without losing the sense of comparison, we just report each buyer's first bid (b_i^5) in case II and III in the table.[8]

In table 2, R and R^j denote the seller's total expected return in case I and each seller's one in case II and III, respectively; R_R and R_R^j denote the corresponding real return in case I and both case II and III, respectively; while π_i and π_i^R denote each buyer's expected gain and real one. For the seller(s), the common reserve price in case II (70.567 RMBs) is higher than the one in case I (70 RMBs), which shows υ_* is increasing in m, and $\lambda_M^{BLR} \geq -\ln\left[1 - F^{n-1}(\upsilon_*)\right]^m$ holds based on Proposition 3.[9] Meanwhile, υ_* is higher in collusion case (75.001 RMBs) than the one in case II, since υ_* is higher than the largest private value among all sellers (60 RMBs) and the average private value is higher than that in case I and II (40 RMBs) by Proposition 10. These results give us an instruction: More intense competition among sellers, higher reserve prices may be possible and feasible. Furthermore, the total expected return of each seller in either case II (44.141 RMBs) or case III (44.141 RMBs) is lower

[7] In the survey of Lucking-Reiley (2000), there are at least three internet auction sites using Dutch auction format, including Intermodal Exchange, Klik-Klok Auctions and Bid.com, though there are much more sites using English ascending-price auction format.

[8] In both case II and III, the winning buyers when there are 4 sellers to 1 seller available are the ones with private values from 99 RMBs to 96 RMBs, and corresponding winning bids are 97.654, 96.924, 95.986, 95.000, respectively.

[9] However, we still have other possible approximate solutions lower than 70 RMBs, e.g. 69.971 RMBs, 69.872 RMBs, and have some higher than 70 but lower than 70.567, e.g. 70.320 RMBs, 70.120 RMBs. Since now $n >> m$ isn't satisfied, or n is not large enough, so the result may only have demonstrative meaning, but not the testing power.

than that in case I (98.020 RMBs) since $\lambda_M^{BLR} \geq -\ln\left[1 - F^{n-1}(\upsilon_*)\right]^m$ by Proposition 3. SInce n =100 is relatively large, then all sellers' total expected returns are hardly different though they have different private values by Proposition 2 and 3.

Table 2: Comparison of the three online auction cases

Cases	Case I			Case II			Case III		
Seller(s)	υ_*	R	R_R	υ_*^j	R^j	R_R^j	υ_*^j	R^j	R_R^j
n =100	70.000	98.020	99.000	70.567	44.141	{95.000,95.986, 96.924,97.654, 97.727}	75.001	44.141	{95.000,95.986, 96.924,97.654, 97.727}
Buyers									
υ_i	b_i	π_i	π_i^R	b_i^5	π_i	π_i^R	b_i^5	π_i	π_i^R
1	0.000	0.0000	0.000	0.000	0.0000	0.000	0.000	0.0000	0.000
...	......	...	...	......	...	...	......	...	...
69	0.000	0.0000	0.000	0.000	0.0000	0.000	0.000	0.0000	0.000
70	70.000	0.0000	0.000	0.000	0.0000	0.000	0.000	0.0000	0.000
71	70.462	1.0126E-15	0.000	70.685	3.0461E-15	0.000	0.000	0.0000	0.000
72	71.323	5.0872E-15	0.000	71.381	2.3494E-14	0.000	0.000	0.0000	0.000
73	72.281	2.1169E-14	0.000	72.295	1.0393E-13	0.000	0.000	0.0000	0.000
74	73.263	8.3463E-14	0.000	73.267	4.1593E-13	0.000	0.000	0.0000	0.000
75	74.251	3.2040E-13	0.000	74.252	1.6001E-12	0.000	0.000	0.0000	0.000
76	75.240	1.2057E-12	0.000	75.240	6.0270E-12	0.000	75.442	4.4243E-12	0.000
...	......	...	...	......	...	...	......	...	...
95	94.050	5.9205E-03	0.000	94.044	0.0294	0.000	94.044	0.0294	0.000
96	95.040	0.0169	0.000	95.023	0.0829	1.000	95.023	0.0829	1.000
97	96.030	0.0476	0.000	95.981	0.2264	1.014	95.981	0.2264	1.014
98	97.020	0.1326	0.000	96.876	0.5807	1.076	96.876	0.5807	1.076
99	98.010	0.3660	0.000	97.561	1.2961	1.346	97.561	1.2961	1.346
100	99.000	1.0000	1.000	97.727	2.2727	2.273	97.727	2.2727	2.273

For buyers, interestingly, the first bids of some buyers with lower private values, namely, from 71 RMBs to 75 RMBs, are smaller when there is only one seller than the ones in case II, while the bids of some higher-private-value buyers with from 77 RMBs to 100 RMBs are bigger in case I than the ones in case II. Such results may be important to reveal the principle that the probability enhancement of winning an auctioned good for more potential auctions may not surely induce bidding buyers' lower bids and higher real gains. On the one hand, some higher-private-value buyers may improve their real gain by lowerly bidding; but on the other hand, some buyers with lower private values may suffer the loss of their real gains. Such results imply that more winning opportunities may make buyers with lower private values feel a little bit higher bid can lead to larger increment of marginal winning probability, so increasing their bids is worthwhile; while such situation may let buyers in the other end perceive that a litte bit lower bid can only entail less decrement of marginal winning probability, thus decreasing their bids is optimal. However, for each buyer who can submit a bid, his expected gain is strictly increasing with more sellers in both case II and III, which looks in contradiction with Proposition 2. But given $\partial v_*/\partial m \leq 0$ is only a relative sufficient condition, $\partial v_*/\partial m > 0$ may also generate the similar results.

6.2 Comparison of Cases with Noncollusion and Collusion

In the heterogeneous reserve prices case with complete information, namely, the noncollusion case, each buyer may possibly bid five times in sequence for potential five auctions. Based on Proposition 9, we can directly calculate each seller's optimal reserve price. Meanwhile, based on Proposition 7, we can also obtain each buyer's bidding strategy and calculate his five sequential bids. It's meaningful to reveal the differencethe of between optimal reserve price in collusion case and the ones in noncollusion case, and to compare the total expected return of each seller as well as the expected gain of each buyer in both cases. Furthermore, we may also demonstrate whether our Conjecture 1 may be correct or not through comparing two settings: n =100 and n =120.

Table 3 demonstrates the results of both sides with n =100 and n =120 in noncollusion case. When n =100, the lowest reserve price is 70 RMBs and the highest one 78.947 RMBs, which makes the optimal reserve price in collusion case lie in the point between these two ones. Interestingly, each seller's expected total return now is higher to a great extent, say, the lowest is 50.1 RMBs and the highest is 72 RMBs, than that, say, 44.141 RMBs, in case III, so is the sum of all sellers' total expected returns.[10] Such comparison may let someone argue that in the simulation settings, noncollusion is more reasonable to be an equilibrium rather than collusion. This argument may be right yet, but when comparing each seller's real total return in both cases, we find out that the real total returns of buyers with private values of 55 RMBs and 60 RMBs are higher in collusion case than the ones in noncollusion case, though the ones of the other converse. Additionally, their real total returns are quite close in both cases. Moreover, since we require $n >> m$ =100 is not high enough, which makes our data possibly fail to reveal the true results. Therefore, either collusion or noncollusion may be an equilibrium, and as a result, both Proposition 9 and 10 are still meaningful. Another interesting finding is that though the seller with the lowest reserve price can have the highest bid in the five auctions, his expected total return may not be highest, namely, R^5 =70 RMBs while R^4 =72 RMBs. Obviously, the

[10]In n 100 case, the program-running results of $\left\{R^j\right\}_{j=3}^{1}$ are 14.946, 7.8087, 4.8196, respectively. However, such results are pseudo ones with the setting of n 100 and m 5, for n is not large enough relative to m since we require $n >> m$ throughout our analysis. In our trials, when $n \geq$ by fixing m, the results of total expected returns of all sellers are consistent with our theory. Hence, for convenience to compare, we take the adjusted but reasonalbe results: 50.100, 55.100, 60.100.

ranks of all sellers may depend on the details of all buyers' private values, all buyer's private values as well as n and m. Hence, that the reason why we don't propose such ranks in our main conclusions.

Table 3: The behaviors of both sides in noncollusion case

Cases	$n=100$							$n=120$					
Sellers	v_*^5	v_*^4	v_*^3	v_*^2	v_*^1	$\{R^j\}_{j=5}^1$	R_R^j	Sellers	v_*^5	v_*^4	v_*^3	v_*^2	v_*^1
	70.000	72.000	74.249	76.575	78.947	{70.000,72.000, 50.100,55.100, 60.100}	{97.680,97.627, 96.911,95.981, 95.000}		69.998	72.079	74.372	76.732	79.122
Buyers								Buyers					
v_i	b_i^5	b_i^4	b_i^3	b_i^2	b_i^1	π_i	π_i^R	v_i	b_i^5	b_i^4	b_i^3	b_i^2	b_i^1
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.83	0.000	0.000	0.000	0.000	0.000
...	...	...	...	...	...	...	...	...	...	...	...	...	...
69	0.000	0.000	0.000	0.000	0.000	0.000	0.000	69.17	0.000	0.000	0.000	0.000	0.000
70	70.000	0.000	0.000	0.000	0.000	0.000	0.000	70.00	70.000	0.000	0.000	0.000	0.000
71	71.000	0.000	0.000	0.000	0.000	0.000	0.000	70.83	70.830	0.000	0.000	0.000	0.000
72	71.510	72.000	0.000	0.000	0.000	7.4026E-15	0.000	71.67	71.660	0.000	0.000	0.000	0.000
73	72.358	72.467	0.000	0.000	0.000	5.1733E-14	0.000	72.50	71.766	70.858	0.000	0.000	0.000
74	73.280	73.307	0.000	0.000	0.000	2.2028E-13	0.000	73.33	72.741	71.73	0.000	0.000	0.000
75	74.374	74.411	74.523	0.000	0.000	9.5248E-13	0.000	74.17	73.745	73.530	0.000	0.000	0.000
76	75.267	75.275	75.304	0.000	0.000	4.0266E-12	0.000	75.00	74.399	73.803	73.989	0.000	0.000
77	76.387	76.417	76.482	76.671	0.000	1.5557E-11	0.000	75.83	75.483	75.316	75.373	0.000	0.000
78	77.254	77.261	77.278	77.330	0.000	6.5262E-11	0.000	76.67	76.102	76.055	76.068	0.000	0.000
79	78.434	78.464	78.517	78.629	78.948	2.1316E-10	0.000	77.50	76.900	76.915	76.947	77.038	0.000
80	79.252	79.259	79.274	79.305	79.400	9.2974E-10	0.000	78.33	77.686	77.690	77.697	77.722	0.000
81	80.192	80.192	80.194	80.201	80.228	3.2714E-09	0.000	79.17	78.690	78.716	78.760	78.852	79.123
82	81.166	81.164	81.163	81.163	81.168	1.0822E-08	0.000	80.00	79.378	79.384	79.396	79.422	79.502
83	82.152	82.149	82.146	82.143	82.142	3.4831E-08	0.000	80.83	80.163	80.164	80.166	80.172	80.195
...	...	...	...	...	...	...	...	...	...	...	...	...	...
95	94.023	94.020	94.017	94.014	94.010	0.0149	0.000	95.83	95.015	95.013	95.011	95.009	95.007
96	95.001	95.000	95.000	95.000	95.000	0.0409	1.000	96.67	95.831	95.832	95.832	95.833	95.833
97	95.956	95.964	95.972	95.981	95.990	0.1124	1.019	97.50	96.629	96.636	96.644	96.652	96.659
98	96.846	96.878	96.911	96.945	96.979	0.3138	1.089	98.33	97.373	97.400	97.428	97.457	97.486
99	97.520	97.627	97.738	97.853	97.969	0.9201	1.373	99.17	97.937	98.027	98.120	98.216	98.312
100	97.680	97.871	98.115	98.448	98.958	2.3197	2.320	100.00	98.072	98.233	98.437	98.714	99.138

For buyers, each bidder improves his expected gain in collusion case relative to that in noncollusion case, except the buyer with the highest private value, and so does the sum of all buyers' expected gains. Remind that the sum of all sellers' total expected returns is lower in collusion case rather than in noncollusion one. Combining these results, we suspect that the total surplus of all buyers, even the surplus of each of them, may not less, even higher when all sellers collude than that when they don't collude with each other., though such conclusion is hard to theoretically discuss. Moreover, by our Conjecture 1, when n is large enough, each buyer at the lowest end who can bid may take the decreasing bidding strategy and each one at the highest end converses. However, when n =100, data in table 3 tell us that each buyer with $\upsilon_i \in \{72, 73, ..., 81\}$, who offers bids in at least two auctions, increasingly bids from the auction of the seller with υ_*^5 =70 RMBs to that of the seller with υ_*^1 =78.947 RMBs, although so does each buyer with $\upsilon_i \geq$97 RMBs. By contrast, when n =120, each buyer with $\upsilon_i \in \{72.50, 73.33, 74.17\}$ takes the decreasing bidding strategy, while the one with $\upsilon_i \geq 96.67$ RMBs takes the reverse strategy. In Addition, there are only not more than 3 transitional buyers who have relatively intermediate private values to bid without monotonicity, and then the higher buyers bid ascendingly again, and then again, we meet one or several transitional buyers, and then the higher buyers bid decreasingly, and after the last one or several transitional ones occur, in the end, the rest of buyers with highest private values submit ascending bids. Hence, based on such complex bidding, we guess our conjecture may be correct to a great extent when n is large enough relative to m.

6.3 The Illustration of Cross-Group Externalities

Our another important focus lies in the illustration of bidder and auctioneer externalities in case II. According to Theorem 1, in our settings, the auctioneer externalities are positive if $\lambda_M^{BLR} \leq -\ln\left[1 - F^{n-1}(\upsilon_*)\right]^m$, and the bidder externalities are positive if $\lambda^{WLR}(\upsilon_*) \geq -\ln F^n(\upsilon_*)$. Since each condition is quite strict sufficient one, and as a whole, the externalities on both sides are very intuitive and obvious, we ignore these constraints when discussing. For demonstrating bidder externalities, we range the number of buyers from 50 to 120; while range the number of sellers from 5 to 20 for auctioneer externalities (see table 4).

Checking out the bidder externalities, it is found that as the number of buyers ascends, each seller's total expected return is increasing, but the speed of such increment decreases. It's natural for the marginal return induced by identical increment of the number of buyers diminishes as n become larger. Thus, to some degree, the data gives us simulative support that the bidder externalities can be positive. By parallel observing, we find that the expected gain of each buyer who bids improves as the number of sellers becomes bigger though m is not small enough relative to n. This indicates that the auctioneer externalities can be also positive for each bidding buyer. Additionally, although the reserve price of each seller is increasing as m ascends, but all bidding buyers' expected gain still enhance for the marginal increment of winning probability accelerates to a great extent.

These results reveal that the positivity of both auctioneer and bidder externalities may hold in looser conditions than that in Theorem 1, even when n is not high enough relative to m. The ubiquity of positive auctioneer and bidder exernalities may provide a simulative evidence for the booming of online auctions.

Table 4: The illustration of cross-group externalities

Bidder	n =50	n =60	n =70	n =80	n =90	n =100	n =110	n =120
externalities	m =5							
R^j	42.657	43.147	43.500	43.766	43.974	44.141	44.278	44.393
Auctioneer	m =5	m =6	m =7	m =8	m =9	m =10	m =15	m =20
externalities	n =100							
v_*^j	70.567	70.712	70.716	70.780	70.781	70.782	70.783	70.784
Buyers(v_i)	π_i	π_i	π_i	π_i	π_i	π_i	π_i	π_i
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
...	...	...	...	...	...	...	...	...
70	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
71	2.5090E-15	2.6512E-15	2.8674E-15	3.0461E-15	3.5849E-15	3.9768E-15	5.3773E-15	7.1699E-15
72	2.3494E-14	2.7106E-14	3.0965E-14	3.5388E-14	4.0658E-14	4.4237E-14	6.6355E-14	8.8474E-14
73	1.0393E-13	1.2381E-13	1.4324E-13	1.6371E-13	1.8571E-13	2.0464E-13	3.0695E-13	4.0927E-13
74	4.1593E-13	4.9716E-13	5.7969E-13	6.6251E-13	7.4573E-13	8.2814E-13	1.2422E-12	1.6563E-12
75	1.6001E-12	1.9188E-12	2.2382E-12	2.5579E-12	2.8782E-12	3.1974E-12	4.7961E-12	6.3948E-12
...	...	...	...	...	...	...	...	...
96	0.0829	0.0990	0.1150	0.1309	0.1466	0.1622	0.2382	0.3110
97	0.2264	0.2685	0.3096	0.3497	0.3889	0.4271	0.6060	0.7661
98	0.5807	0.6756	0.7648	0.8488	0.9280	1.0028	1.3227	1.5743
99	1.2961	1.4499	1.5848	1.7045	1.8119	1.9089	2.2885	2.5610
100	2.2727	2.4370	2.5776	2.7004	2.8095	2.9076	3.2884	3.5610

7 Conclusion and Open Questions

The purpose of this paper is to capture the important features of online auctions other than traditional auctions, namely, competition among sellers and cross-group externalities, and explore corresponding effects on buyers' equilibrium bidding strategies, setting of sellers' optimal reserve prices, and the expected returns of both sides. Since we focus on the SIPV model, our paper can be viewed as a competitive-seller extension of Myerson (1981), Riley and Samuelson (1981), and fits for two main auction formats which have been discussed in Riley and Samuelson (1981), including high bid (Dutch) auction and English auction.

We investigate two categories of typical online auction cases: sellers with identical private values and with different ones. In the former category, when buyers can multiauction, that is, submit bids in all auctions if possible, all sellers set the common optimal reserve price. Surprisingly, such common reserve price is decreasing as the number of buyers n becomes larger, and may ascend as the number of sellers m

increases if the bidding likelihood ratio λ_M^{BLR} of the buyer with private value equal to the reserve price is high enough, though vice versa. When λ_M^{BLR} is high enough, each seller's total expected return is strictly decreasing as sellers' competition turns to be more intense. Moreover, some part of buyers in the lower end may bid more aggressively than they do in one-seller case, and each bidding buyer's expected gain is strictly increasing in m if λ_M^{BLR} is high enough, but may not be improved without any constraint. By contrast, when all buyers can bid in only one auction, we have the less buyers version for each seller of one-seller auction. So naturally, the common equilibrium reserve price is the same as the one with one seller, but each seller's total expected return is decreasing in m only when the winning probability of a buyer with private value equal to the reserve price is large enough by Proposition 5, and is strictly increasing in n with more strict constraints.

Meanwhile, there do endogenously exist cross-group externalities in online auctions according to Theorem 1 and 2, which is quite important and distinguishes from traditional auctions. In bidding by multiauctioning and single-auctioning cases, both the auctioneer and bidder externalities can be positive if some sufficient conditions, may be too strict, are satisfied, which provides the theoretical evidence for the rapid development of online auctions in the last decade.

In the latter category, when the information of the sellers' private values is symmetric to all of them, there may be two reasonable equilibria: noncollusion and collusion. In noncollusion case, the sellers design different reserve prices, namely, larger is a seller's private value, higher is his reserve price. Then for each potential bidding buyer, he takes sequential bidding strategy which may be descending from the first auction to the last one when he has quite low private value, while ascending when he has quite high private value by Conjecture 1. Hence, the equilibrium in this case is a sequential one. However, in collusion case, all sellers set a common but higher reserve price just like bidding by multiauctioning case in the first category, and so do the buyers. When information is asymmetric, the design of reserve price of each seller is an complex extension of that in symmetric information, and the reserve prices are surely different again. Because of this, the bidding strategy for each buyer is the same as that in noncollusion case.

Furthermore, by comparing almost all of above cases with complete information, our simple simulation demonstrate and support our main conclusions. Most importantly, it demonstrates whether the auctioneer and bidder externalities are positive and how they work. Additionally, it also helps to preliminarily test our conjecture about the buyers' bidding strategies in noncollusion case, and give a simulative support.

Since the online auctions are much more complicated than traditional ones, our model is too restrictive to reflect all of the uique but important features of online auctions though conveniently tractable. Therefore, we still have quite a lot open questions to study, including the follows.

7.1 Reauctioning

In fact, empirical evidences indicate that many sellers reauction their goods by internet auction houses or platforms, say, eBay, Overstock, Ubid, and may do so multiple times. When there is only one seller, resale situation has been discussed by a series of literatures without considering auction house (McAfee and Vincent, 1997; Horstmann and LaCasse, 1997; Zheng, 2002), and with considering internet auction house (Matros, Zapechelnyuk, 2008a; Matros, Zapechelnyuk, 2008b).

In our paper, we just capture each seller's one-shot auction case, which is restrictive to some extent. When we permit each seller to reauction his good, the design of reserve price for each seller turns to be a recursive optimality problem in which each seller expects a reauction probability in every round. However, in the case that the sellers have heterogenous private values, the analysis will become too much complicated. Hence, our future work may focus on the case that all sellers have homogenous private values, which does make sense though is still somewhat restrictive.

7.2 Effects of Online Auction Platform

If we investigate the online auctions from the perspective of two-sided market, the behaviors of online auction platform may be crucial for the result of each online auction and the equilibrium behaviors of both buyers and sellers. Intuitively, what the online auction platform charges to online auction participants will definitely affect the transaction prices as well as the numbers of both sides, and subseqently affect the benefits of both sides by cross-group externalities and the change of the number of potential trade opportunities. Therefore, a general model describing online auctions should incorporate the behavior of online auction platform and consider the effects of the fees it charges.

In essence, Matros and Zapechelnyuk (2008a, 2008b) have discussed the optimal fees of online mediator in the one-seller case without considering the cross-group externalities. By comparison with the literature of two-sided market, the listing fee in their papers may be viewed as the membership fee, and the closing fee as the usage fee. They conclude that the optimal listing fee is zero. In terms of two-sded market, the sellermay only have the usage externalities if such externalities exist in their papers.[11] However, the buyers can't enjoy the autioneer externalities and as a result, the auction mediator can't endogeoously determine the number of each side by charging fees, which makes their papers restrictive to capture the two-sided market feature of online auction. Hence, the problem of desining the optimal fees for an online auction when there are competitive sellers does be a challenge for further study.

[11] Rochet and Tirole (2003) discusses the pricing problem of competitive platforms when only considering the usage externalities.

7.3 Correlation of Bidders' Valuations

Whether the affiliation effects happen may depend on the category of goods to a great extent. One the one hand, for the unstandard goods which can be characterized by value uncertainty, unfamiliarity, too much high value, or with historical meaning, the private values of buyers may be much harder affiliated, such as antiques, souvenirs, and goods with folk features. On the other hand, for standard goods which can be captured by value certainty, familiarity, not too much high value or necessity, the affiliation effects may happen much easily, e.g., fast moving consumer goods, electronical goods (TV, refrigerator, Telephone, PC), and digital goods (Softwares, Flash discs, MP3 and MP4 players, ipod). Such comparison is quite intuitive, which is consistent with people's consumption experiences, and can explain why traditional auctions almost only focus on unstandard goods. In the online auctions, in many cases, the bidders' valuations are affiliated since many standard goods are auctioned. For instance, on eBay, the listing goods include the following standard ones: computer softvare, electronics, toys and trading cards, even the used equipment.

In the literatures of traditional auctions, the "Affiliated Private Value" Models were proposed to investigate the equilibrium behaviors of both buyers and selers in such cases (Milgrom and Weber, 1982; Pinkse and Tan, 2005). However, such models involve only one seller, which can't reflect the important teatures of online auctions. For both theoretical and practical needs, we should extend these models to multiple sellers.

7.4 Collusion among Sellers

In both traditional and online auctions, especially the former, the collusion among buyers is not a rare phenomenon. The three main online auction platforms eBay, Yahoo, and Amazon and other famous ones ususally apply English auction format, which makes the buyers reatively easily to achieve collusion from the empirical evidences by jump bidding (Sherstyuk, 2002; Brusco and Lopomo, 2002; Kwasnica and Sherstyuk, 2007), sniping (Ockenfels and Roth, 2002, 2006; Wintr, 2008), and withholding bid.

However, the above collusion strategies can be seldomly effective and feasible in online auction practice sicne there may be thousands of buyers who come from different provinces or states, even different countries, and in many cases, they are anonymous. For instance, Overstock has 7.6 millions U.S. monthly users on June 30th and 14.6 millions, the maximal number, on January 2nd, 2009. On the other side, the number of the sellers for the similar goods may be relatively small, ranging from about tens to hundreds. For example, on April, 2009, there are only about 30-50 sellers auctioning American Eagle Silver Dollar on Overstock. So the sellers may be much more easily and do have incentives to form collusion though they may have different private values. Proposition 10 also indicates that such collusion may be an equilibirum for each seller. Therefore, investigating collusion among sellers other than buyers in online auctions should be more meaningful.

7.5 Learning of Buyers

Another important question is the learning effects of both buyers and sellers, especially the former, in online auctions. For buyers, there are three factors inducing such question: duration, reauction and competition among sellers.

These three factors may commonly make buyers learn in the process of an online auction if allowing buyers' multiauctioning. When each seller's auction last for a period of time, say, 7 days, for the case with heterogeneous reserve prices, the buyers may learn about the distribution of the other bidders' private values from the previous auction(s). By some information updating rule, e.g., Bayesian rule, buyers can reschedule their sequential bidding strategies with updated information to the better ones. Even in the single-auctioning case, the last two factors may contribute to each buyer's learning similarly. Such learning may influence the behaviors of both sides, even the design of both auction formats and charging mechanism of the online auction platform, which may be another important feature of online auctions.

Appendix

Proof of Proposition 1. When there are m sellers, for buyer i, probability that he wins with his report x is

$$1-\left[1-F^{n-1}(x)\right]^m.$$

Then his expected gain is

$$\pi(x,\upsilon_i)=\upsilon_i\left[1-\left[1-F^{n-1}(x)\right]^m\right]-P_i(x), \tag{A1}$$

where $P_i(x)$ is his expected payment. Based on optimal mechanism design, buyer i's optimal report must be to select $x=\upsilon_i$ with bid $b(\upsilon_i)$, which must satisfy the FOC of $\pi(x,\upsilon_i)$:

$$\left.\frac{\partial\pi\left(x,\upsilon_i\right)}{\partial x}\right|_{x=\upsilon_i}=\upsilon_i\frac{d\left[1-\left[1-F^{n-1}(\upsilon_i)\right]^m\right]}{d\upsilon_i}-P_i'(\upsilon_i)=0.$$

Hence, with the buyer's private value υ_*, we have the boundary condition:

$$\upsilon_*\left[1-\left[1-F^{n-1}(\upsilon_*)\right]^m\right]-P_*(\upsilon_*)=0. \tag{A2}$$

Given $\upsilon_i\geq\upsilon_*$, buyer i's υ_i must satisfy the following differential equation

$$\upsilon_i\frac{d\left[1-\left[1-F^{n-1}(\upsilon_i)\right]^m\right]}{d\upsilon_i}=P_i'(\upsilon_i)\ \forall\upsilon_i\geq\upsilon_*. \tag{A3}$$

Solving (A3) by using the boundary condition (A2), we have buyer i's expected payment as

$$P_i(\upsilon_i)=\upsilon_i\left[1-\left[1-F^{n-1}(\upsilon_i)\right]^m\right]-\int_{\upsilon_*}^{\upsilon_i}\left[1-\left[1-F^{n-1}(x)\right]^m\right]dx\ \forall\upsilon_i\geq\upsilon_* \tag{A4}$$

Since each seller knows F, his expected revenue is

$$\begin{aligned}\overline{P_i}&=\int_{\upsilon_*}^{\overline{\upsilon}}P_i(\upsilon_i)f(\upsilon_i)d\upsilon_i\\&=\int_{\upsilon_*}^{\overline{\upsilon}}\left[\upsilon f\left(\upsilon\right)+F(\upsilon)-1\right]\left[1-\left[1-F^{n-1}(\upsilon)\right]^m\right]d\upsilon.\end{aligned}$$

Since the equilibrium is symmetric for all buyers, the sum of all seller's expected revenue is , and each seller's is $\frac{n\overline{P_i}}{m}$. ■

Proof of Proposition 2 . In high bid auction, for buyer i, when there are j auctions available, his expected payment is

$$P_i^j(\upsilon_i)=b^j(\upsilon_i)\left[1-\left[1-F^{n-m+j-1}(\upsilon_i)\right]^j\right].$$

Then by using (A4), we can directly obtain buyer i's equilibrium bidding strategy:

$$b^j(v_i) = v_i - \frac{\int_{v_*}^{v_i} \left[1 - [1 - F^{n-m+j-1}(x)]^j\right] dx}{1 - [1 - F^{n-m+j-1}(v_i)]^j} \; \forall j \in \{1, 2, ..., m\}$$

for the boundary conditions

$$v_* \left[1 - \left[1 - F^{n-m+j-1}(v_*)\right]^j\right] - P_*^j(v_*) = 0 \; \forall j \in \{1, 2, ..., m\}$$

are still satisfied at $v_i = v_*$. Moreover, by (A1), buyer i's expected gain is

$$\pi(v_i, v_i) = \int_{v_*}^{v_i} \left[1 - \left[1 - F^{n-1}(x)\right]^m\right] dx.$$

Thus differentiating $\pi(v_i, v_i)$ with respect to m, we have

$$\frac{\partial \pi(v_i, v_i)}{\partial m} = -\left[1 - \left[1 - F^{n-1}(v_*)\right]^m\right] \frac{\partial v_*}{\partial m} - \int_{v_*}^{v_i} \left[1 - F^{n-1}(x)\right]^m \ln\left[1 - F^{n-1}(x)\right] dx > 0$$

if $\frac{\partial v_*}{\partial m} \leq 0$, which indicates that $\pi(v_i, v_i)$ is strictly increasing in m. ■

Proof of Proposition 3. For each seller, the probability that he retains his good after the auction is $F^n(v_*)$. Then by Proposition 1, his total expected return is

$$R(v_0, v_*) = v_0 F^n(v_*) + \frac{n}{m} \int_{v_*}^{\overline{v}} \left[v f(v) + F(v) - 1\right] \left[1 - \left[1 - F^{n-1}(v)\right]^m\right] dv. \quad \text{(A5)}$$

Differentiating (A5) with respect to v_* and by FOC, we have

$$v_* - \frac{1 - F(v_*)}{f(v_*)} = \frac{m F^{n-1}(v_*)}{1 - [1 - F^{n-1}(v_*)]^m} v_0 \equiv \overline{V}(m), \quad \text{(A6)}$$

which describes the optimal reserve price. Thus,

$$\frac{\partial \overline{V}(m)}{\partial m} = \frac{F^{n-1}(v_*) \left[1 - [1 - m \ln[1 - F^{n-1}(v_*)]] \left[1 - F^{n-1}(v_*)\right]^m\right]}{\left[1 - [1 - F^{n-1}(v_*)]^m\right]^2} v_0 \leq 0$$

If

$$\lambda_M^{BLR} \equiv \frac{1 - [1 - F^{n-1}(v_*)]^m}{[1 - F^{n-1}(v_*)]^m} \leq -\ln\left[1 - F^{n-1}(v_*)\right]^m.$$

Then by Myerson's regular condition, $v_* - \frac{1 - F(v_*)}{f(v_*)}$ is strictly increasing, thus v_* is decreasing in m. Similarly, if

$$\lambda_M^{BLR} \geq -\ln\left[1 - F^{n-1}(v_*)\right]^m,$$

then is v_* increasing in m.

Furthermore, differentiating (A5) with respect to m yields

$$\begin{aligned}
\frac{\partial R(\upsilon_0,\upsilon_*)}{\partial m} &= n\upsilon_0 F^{n-1}(\upsilon_*)f(\upsilon_*)\frac{\partial \upsilon_*}{\partial m} - \frac{n}{m^2}\int_{\upsilon_*}^{\overline{\upsilon}} \left[\upsilon f\left(\upsilon\right)+F(\upsilon)-1\right]\left[1-\left[1-F^{n-1}(\upsilon)\right]^m\right] d\upsilon \\
& -\frac{n}{m} f(\upsilon_*)\left[\upsilon_* - \frac{1-F(\upsilon_*)}{f(\upsilon_*)}\right]\left[1-\left[1-F^{n-1}(\upsilon_*)\right]^m\right]\frac{\partial \upsilon_*}{\partial m} \\
& -\frac{n}{m}\int_{\upsilon_*}^{\overline{\upsilon}} \left[\upsilon f\left(\upsilon\right)+F(\upsilon)-1\right]\left[1-F^{n-1}(\upsilon)\right]^m \ln\left[1-F^{n-1}(\upsilon)\right] d\upsilon \\
&= -\frac{n}{m}\int_{\upsilon_*}^{\overline{\upsilon}} \left[\upsilon f\left(\upsilon\right)+F(\upsilon)-1\right]\left[\begin{array}{c} \frac{1-\left[1-F^{n-1}(\upsilon)\right]^m}{m}+ \\ \left[1-F^{n-1}(\upsilon)\right]^m \ln\left[1-F^{n-1}(\upsilon)\right] \end{array}\right] d\upsilon
\end{aligned}$$

since υ_* satisfies (A6). Define

$$\widetilde{R}(\upsilon) \equiv \frac{1-\left[1-F^{n-1}(\upsilon)\right]^m}{m} + \left[1-F^{n-1}(\upsilon)\right]^m \ln\left[1-F^{n-1}(\upsilon)\right].$$

Then

$$\frac{\partial \widetilde{R}(\upsilon)}{\partial \upsilon} = -m(n-1)\left[1-F^{n-1}(\upsilon)\right]^{m-1} F^{n-2}(\upsilon)f(\upsilon)\ln\left[1-F^{n-1}(\upsilon)\right] \geq 0 \ \forall \upsilon \in \left[\upsilon_*, \overline{\upsilon}\right].$$

Thus, if $\widetilde{R}(\upsilon_*) \geq 0$, by Myerson's regular condition, $\frac{\partial R(\upsilon_0,\upsilon_*)}{\partial m} < 0$. ■

Proof of Corollary 1. Based on (A6), we define $\widetilde{V}(n) \equiv \overline{V}(m)$. Thus, we have

$$\frac{\partial \widetilde{V}(n)}{\partial n} = \frac{F^{n-1}(\upsilon_*)\left[\ln F(\upsilon_*)\right]\left[1-\left[\left[1-F^{n-1}(\upsilon_*)\right]^m + mF^{n-1}(\upsilon_*)\left[1-F^{n-1}(\upsilon_*)\right]^{m-1}\right]\right]}{\left[1-\left[1-F^{n-1}(\upsilon_*)\right]^m\right]^2} m\upsilon_0.$$

Since $\sum_{i=0}^m C_m^i \left[F^{n-1}(\upsilon_*)\right]^i \left[1-F^{n-1}(\upsilon_*)\right]^{m-i} = 1$ for a buyer whose private value is υ_* and $\upsilon_*(\underline{\upsilon}) > \underline{\upsilon}$, it can easily yield that $\left[1-F^{n-1}(\upsilon_*)\right]^m + mF^{n-1}(\upsilon_*)\left[1-F^{n-1}(\upsilon_*)\right]^{m-1} < 1$. Therefore, $\frac{\partial \widetilde{V}(n)}{\partial n} \leq 0$. Then by Myerson's regular condition, υ_* is decreasing in n, and is strictly decreasing if $\upsilon_* < \overline{\upsilon}$.

Furthermore, by (A5), we have

$$\begin{aligned}
\frac{\partial R(\upsilon_0,\upsilon_*)}{\partial n} &= f(\upsilon_*)\left[\upsilon_0 F^n(\upsilon_*)\ln F(\upsilon_*) - \frac{n}{m}\left[\upsilon_* - \frac{1-F(\upsilon_*)}{f(\upsilon_*)}\right]\left[1-\left[1-F^{n-1}(\upsilon_*)\right]^m\right]\right]\frac{\partial \upsilon_*}{\partial n} \\
& +\frac{1}{m}\int_{\upsilon_*}^{\overline{\upsilon}} \left[\upsilon f\left(\upsilon\right)+F(\upsilon)-1\right]\left[\begin{array}{c} 1-\left[1-F^{n-1}(\upsilon)\right]^m + \\ nm\left[1-F^{n-1}(\upsilon)\right]^{m-1} F^{n-1}(\upsilon)\ln F(\upsilon) \end{array}\right] d\upsilon.
\end{aligned}$$

Then by definition of $\lambda^{WLR}(\upsilon)$,

$$\lambda^{WLR}(\upsilon) = \frac{1-\left[1-F^{n-1}(\upsilon)\right]^m}{m\left[1-F^{n-1}(\upsilon)\right]^{m-1} F^{n-1}(\upsilon)} \ \forall \upsilon \in \left[\upsilon_*, \overline{\upsilon}\right].$$

Differentiating $\lambda^{WLR}(\upsilon)$ with respect to υ yields

$$\frac{\partial \lambda^{WLR}(\upsilon)}{\partial \upsilon}=\frac{(n-1)f(\upsilon)F^{n-2}(\upsilon)\left[1-F^{n-1}(\upsilon)\right]^{m-2}\left[mF^{n-1}(\upsilon)+\left[1-F^{n-1}(\upsilon)\right]^{m}-1\right]}{m\left[\left[1-F^{n-1}(\upsilon)\right]^{m-1}F^{n-1}(\upsilon)\right]^{2}}.$$

For convenience, define

$$\widehat{R}(\upsilon)\equiv mF^{n-1}(\upsilon)+\left[1-F^{n-1}(\upsilon)\right]^{m}-1.$$

Then we can derive

$$\frac{\partial \widehat{R}(\upsilon)}{\partial \upsilon}=m(n-1)f(\upsilon)F^{n-2}(\upsilon)\left[1-\left[1-F^{n-1}(\upsilon)\right]^{m-1}\right]>0$$

since $m\geq 2$ and $\upsilon\geq\upsilon_*>\underline{\upsilon}$. As a result, $\widehat{R}(\upsilon)>0$ $\forall\upsilon\in[\upsilon_*,\overline{\upsilon}]$ for $\widehat{R}(\underline{\upsilon})=0$. Hence, we obtain $\frac{\partial\lambda^{WLR}(\upsilon)}{\partial\upsilon}>0$. Meanwhile, $-\ln F^{n}(\upsilon)$ is decreasing in υ. Therefore, $R(\upsilon_0,\upsilon_*)$ is strictly increasing in n if $\lambda^{WLR}(\upsilon_*)\geq-\ln F^{n}(\upsilon_*)$ since $\frac{\partial\upsilon_*}{\partial n}\leq 0$. ■

Proof of Proposition 5. By parallel analysis with the case with multiauctioning, each buyer's expected gain in equilibrium is

$$\pi(\upsilon_i,\upsilon_i)=\int_{\upsilon_*}^{\upsilon_i}F^{n/m-1}(x)dx.$$

Differentiating $\pi(\upsilon_i,\upsilon_i)$ with respect to m yields

$$\frac{\partial\pi(\upsilon_i,\upsilon_i)}{\partial m}=-\frac{n}{m^2}\int_{\upsilon_*}^{\upsilon_i}F^{n/m-1}(x)\ln F(x)dx>0.$$

Moreover, based on Proposition 4, each seller's total expected return is

$$R(\upsilon_0,\upsilon_*)=\upsilon_0F^{n/m}(\upsilon_*)+\frac{n}{m}\int_{\upsilon_*}^{\overline{\upsilon}}\left[\upsilon f(\upsilon)+F(\upsilon)-1\right]F^{n/m-1}(\upsilon)d\upsilon.$$

Thus, we have

$$\begin{aligned}\frac{\partial R(\upsilon_0,\upsilon_*)}{\partial m} &= -\upsilon_0\frac{n}{m^2}F^{n/m}(\upsilon_*)\ln F(\upsilon_*)\\ &\quad -\frac{n}{m^2}\int_{\upsilon_*}^{\overline{\upsilon}}\left[\upsilon f(\upsilon)+F(\upsilon)-1\right]\left[1+\frac{n}{m}\ln F(\upsilon)\right]F^{n/m-1}(\upsilon)d\upsilon,\end{aligned}$$

which is positive if $1+\ln F^{n/m}(\upsilon_*)\geq 0$. Meanwhile, we also have

$$\begin{aligned}\frac{\partial R(\upsilon_0,\upsilon_*)}{\partial n} &= \frac{\upsilon_0}{m}F^{n/m}(\upsilon_*)\ln F(\upsilon_*)+\frac{1}{m}\int_{\upsilon_*}^{\overline{\upsilon}}\left[\upsilon f(\upsilon)+F(\upsilon)-1\right]F^{n/m-1}(\upsilon)d\upsilon\\ &\quad +\frac{n}{m^2}\int_{\upsilon_*}^{\overline{\upsilon}}\left[\upsilon f(\upsilon)+F(\upsilon)-1\right]F^{n/m-1}(\upsilon)\ln F(\upsilon)d\upsilon.\end{aligned}$$

Then based on (8), given $1+\ln F^{n/m}(\upsilon_*)>0$, substituting υ_0 by $\upsilon_*-\frac{1-F(\upsilon_*)}{f(\upsilon_*)}$ and rearranging the above derivative yields

$$\begin{aligned}\frac{\partial R(\upsilon_0,\upsilon_*)}{\partial n} &= \frac{1}{m}\left[\begin{array}{c}\int_{\upsilon_*}^{\overline{\upsilon}}\left[\upsilon_*-\frac{1-F(\upsilon_*)}{f(\upsilon_*)}\right]\frac{\ln F(\upsilon_*)}{1-F(\upsilon_*)}F^{n/m}(\upsilon_*)f(\upsilon)\,d\upsilon+ \\ \int_{\upsilon_*}^{\overline{\upsilon}}\left[\upsilon-\frac{1-F(\upsilon)}{f(\upsilon)}\right]\frac{1+\ln F^{n/m}(\upsilon)}{F(\upsilon)}F^{n/m}(\upsilon)f(\upsilon)\,d\upsilon\end{array}\right] \\ &> \frac{1}{m}\left[\int_{\upsilon_*}^{\overline{\upsilon}}\left[\upsilon_*-\frac{1-F(\upsilon_*)}{f(\upsilon_*)}\right]\left[\frac{\ln F(\upsilon_*)}{1-F(\upsilon_*)}+\frac{1+\ln F^{n/m}(\upsilon_*)}{F(\upsilon_*)}\right]F^{n/m}(\upsilon_*)f(\upsilon)\,d\upsilon\right]\end{aligned}$$

since Myerson's regular condition holds and $\frac{\left[1+\ln F^{n/m}(\upsilon)\right]F^{n/m}(\upsilon)}{F(\upsilon)}$ is strictly increasing in υ. Hence, $\frac{\ln F(\upsilon_*)}{1-F(\upsilon_*)}+\frac{1+\ln F^{n/m}(\upsilon_*)}{F(\upsilon_*)}\geq 0$ if

$$\frac{1-F(\upsilon_*)}{F(\upsilon_*)}\geq\frac{-\ln F(\upsilon_*)}{1+\ln F^{n/m}(\upsilon_*)},$$

which results in $\frac{\partial R(\upsilon_0,\upsilon_*)}{\partial n}>0$. ■

Proof of Proposition 9. Based on (27), the FOC condition of $R^j\left(\upsilon_0^j;\left\{\upsilon_*^h\right\}_{h=m}^j\right)$ with respect to υ_*^j $\forall j\in\{1,2,...,m\}$ can be yielded as

$$\begin{aligned}\frac{\partial R^j\left(\upsilon_0^j;\left\{\upsilon_*^h\right\}_{h=m}^j\right)}{\partial\upsilon_*^j} &= (n-m+j)\upsilon_0^jF^{n-m+j-1}(\upsilon_*^j)f(\upsilon_*^j)\Pi_{k=0}^{m-j-1}\left[1-F^{n-k}(\upsilon_*^{m-k})\right]+ \\ &\quad (n-m+j)\left[\begin{array}{c}\int_{\upsilon_*^j}^{\overline{\upsilon}}\left[\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon)\right]\right]F^{n-m+j-1}(\upsilon_*^j)f(\upsilon)\,d\upsilon \\ -\upsilon_*^jF^{n-m+j-1}(\upsilon_*^j)f(\upsilon_*^j)\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon_*^j)\right]\end{array}\right] \\ &= 0.\end{aligned}$$

Hence, we have

$$\upsilon_*^j=\upsilon_0^j\frac{\Pi_{k=0}^{m-j-1}\left[1-F^{n-k}(\upsilon_*^{m-k})\right]}{\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon_*^j)\right]}+\frac{\int_{\upsilon_*^j}^{\overline{\upsilon}}\left[\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon)\right]\right]f(\upsilon)d\upsilon}{\left[\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon_*^j)\right]\right]f(\upsilon_*^j)},$$

which is dependent of the number of all buyers and $\left\{\upsilon_*^h\right\}_{h=m}^{j+1}$. ■

Proof of Proposition 10. Since we don't permit sellers to communicate with each other before they set their reserve prices, in terms of equilibrium refinement, there are two most possible equilibria: all sellers fully uncooperate and fully collude, though there may be other possible equilibria that require quite complex conditions.

Our proof focuses on the idea that given $\upsilon_*^M\geq\upsilon_{01}>\upsilon_{02}>...>\upsilon_{0m}$, if each seller doesn't have the incentive to deviate from the noncooperation result to the collusion result, then the

former does be an equilibrium. We consider the proximate case that under the result with different reserve prices, each seller has the same number of buyers, i.e., n

buyers. Thus, based on (27), we obtain a typical seller j's total expected return of

$$R^j\left(\upsilon_0^j;\{\upsilon_*^h\}_{h=m}^j\right) = \upsilon_0^j\left[1-\Pi_{k=0}^{m-j}\left[1-F^n(\upsilon_*^{m-k})\right]\right]+ n\int_{\upsilon_*^j}^{\overline{\upsilon}}\left[\Pi_{k=1}^{m-j}\left[1-F^n(\upsilon)\right]\right]\left[\upsilon F^{n-1}(\upsilon)-\int_{\upsilon_*^j}^{\upsilon}F^{n-1}(x)dx\right]f(\upsilon)\,d\upsilon. \quad (A7)$$

Meanwhile, by (A5), a typical seller's total expected return with υ_*^M is

$$R\left(\upsilon_0^j,\upsilon_*^M\right)=\upsilon_0^jF^n(\upsilon_*^M)+n\int_{\upsilon_*^M}^{\overline{\upsilon}}\left[\upsilon f(\upsilon)+F(\upsilon)-1\right]\frac{\left[1-\left[1-F^{n-1}(\upsilon)\right]^m\right]}{m}d\upsilon. \quad (A8)$$

For convenient comparison, for seller with υ_*^m, we have

$$\begin{aligned}R^m\left(\upsilon_0^m;\upsilon_*^m\right) &= \upsilon_0^mF^n(\upsilon_*^m)+n\int_{\upsilon_*^m}^{\overline{\upsilon}}\left[\upsilon F^{n-1}(\upsilon)-\int_{\upsilon_*^m}^{\upsilon}F^{n-1}(x)dx\right]f(\upsilon)\,d\upsilon \quad (A9)\\ &= \upsilon_0^mF^n(\upsilon_*^m)+n\int_{\upsilon_*^m}^{\overline{\upsilon}}\left[\upsilon f(\upsilon)+F(\upsilon)-1\right]F^{n-1}(\upsilon)d\upsilon.\end{aligned}$$

For the same seller with υ_0^m, when $\upsilon_*^M=\upsilon_*^m$ and $m=1$, $R\left(\upsilon_0^m;\upsilon_*^M\right)=R^m\left(\upsilon_0^m;\upsilon_*^m\right)$. Thus, if $m>1$, $R\left(\upsilon_0^m;\upsilon_*^M\right)\leq R^m\left(\upsilon_0^m;\upsilon_*^m\right)$ since $R\left(\upsilon_0^m;\upsilon_*^x\right)$ is maximized at $\upsilon_*^x=\upsilon_*^m$. Moreover, by Proposition 3, if $\lambda_M^{BLR}\geq-\ln\left[1-F^{n-1}(\upsilon_*^M)\right]^m$, $R\left(\upsilon_0^m;\upsilon_*^M\right)$ is strictly decreasing in m. Therefore, $R\left(\upsilon_0^m;\upsilon_*^M\right)<R^m\left(\upsilon_0^m;\upsilon_*^m\right)$ holds, as a result, seller m would not deviate from υ_*^m. However, if $\lambda_M^{BLR}<-\ln\left[1-F^{n-1}(\upsilon_*^M)\right]^m$, $R\left(\upsilon_0^m;\upsilon_*^M\right)$ may not be decreasing in m, so $R\left(\upsilon_0^m;\upsilon_*^M\right)\geq R^m\left(\upsilon_0^m;\upsilon_*^m\right)$ may hold. Furthermore, by (A7), (A8) and (A9), if $\lambda_M^{BLR}>-\ln\left[1-F^{n-1}(\upsilon_*^j)\right]^m$, we have

$$\begin{aligned}R(\upsilon_0^j,\upsilon_*^j) &= \upsilon_0^jF^n(\upsilon_*^j)+n\int_{\upsilon_*^j}^{\overline{\upsilon}}\left[\upsilon f(\upsilon)+F(\upsilon)-1\right]\frac{\left[1-\left[1-F^{n-1}(\upsilon)\right]^m\right]}{m}d\upsilon\\ &< R\left(\upsilon_0^j;\{\upsilon_*^h\}_{h=m}^j\right)=\upsilon_0^jF^n(\upsilon_*^j)+n\int_{\upsilon_*^j}^{\overline{\upsilon}}\left[\upsilon F^{n-1}(\upsilon)-\int_{\upsilon_*^j}^{\upsilon}F^{n-1}(x)dx\right]f(\upsilon)\,d\upsilon.\end{aligned}$$

However, $1-\Pi_{k=0}^{m-j}\left[1-F^n(\upsilon_*^{m-k})\right]\geq F^n(\upsilon_*^j)$ may hold if m is large enough. Meanwhile,

$$\begin{aligned}&n\int_{\upsilon_*^j}^{\overline{\upsilon}}\left[\Pi_{k=1}^{m-j}\left[1-F^n(\upsilon)\right]\right]\left[\upsilon F^{n-1}(\upsilon)-\int_{\upsilon_*^j}^{\upsilon}F^{n-1}(x)dx\right]f(\upsilon)\,d\upsilon\\ \geq\ &n\int_{\upsilon_*^j}^{\overline{\upsilon}}\left[\upsilon f(\upsilon)+F(\upsilon)-1\right]\frac{\left[1-\left[1-F^{n-1}(\upsilon)\right]^m\right]}{m}d\upsilon\end{aligned}$$

may hold if n is large enough and $n>>m$, which results in $R(\upsilon_0^j,\upsilon_*^j)<R^j\left(\upsilon_0^j;\{\upsilon_*^h\}_{h=m}^j\right)$. Finally, Assume that υ_{*j}^M is the optimal reserve price when all sellers' private values are υ_{0j}, we can directly obtain $R\left(\upsilon_0^j,\upsilon_*^j\right)\leq R\left(\upsilon_0^j,\upsilon_{*j}^M\right)$. Based on the above

analysis, when $\lambda_M^{BLR} > -\ln\left[1-F^{n-1}(\upsilon_{*j}^M)\right]^m$, $n >> m$, and m is large enough, $R(\upsilon_0^j, \upsilon_{*j}^M) \leq R^j\left(\upsilon_0^j; \{\upsilon_*^h\}_{h=m}^j\right)$ may hold. Since $R\left(\upsilon_0^j, \upsilon_*^M\right) \leq R\left(\upsilon_0^j, \upsilon_{*j}^M\right)$ by the definition of υ_{*j}^M, $R(\upsilon_0^j, \upsilon_{*j}^M) \leq R^j\left(\upsilon_0^j; \{\upsilon_*^h\}_{h=m}^j\right)$ may also hold. By our basic assumption, $n >> m$, hence $n-m+j-1 \approx n-m+j \approx n$, which results in that seller j may not deviate from υ_*^j $\forall j \in \{1, 2, ..., m-1\}$.

By parallel analysis, all sellers may not deviate from the collusion result since it has the largest joint profit.

For the setting of common optimal reserve price, by the method in the proof of Proposition 3, we can similarly obtain υ_*^M and $\frac{\partial \upsilon_*^M}{\partial m} < 0$. ■

Proof of Proposition 12. By (27) and (30), $\forall l \in \{1, 2, ..., m\}$, seller l's ex ante total expected return can be expressed by

$$E_{\upsilon_*^{-l}} R^l(\upsilon_{0l}; \upsilon_*^l, \upsilon_*^{-l}) = \sum_{j=m}^{1} \left[C_{m-1}^{j-1}\left[1-F_s(\upsilon_*^l)\right]^{j-1} F_s^{m-j}(\upsilon_*^l)\right] R^j\left(\upsilon_{0l}; \{\upsilon_*^h\}_{h=m}^j\right).$$

By FOC condition of $E_{\upsilon_*^{-l}} R^l(\upsilon_{0l}; \upsilon_*^l, \upsilon_*^{-l})$ with respect to υ_*^l, we have

$$\upsilon_{0l}\left[\begin{array}{c} f_s(\upsilon_*^l)\sum_{j=m}^{1}\left[\frac{m-j}{F_s(\upsilon_*^l)} - \frac{j-1}{1-F_s(\upsilon_*^l)}\right] P_s\left(\upsilon_*^l = \upsilon_*^j\right) P^j(\upsilon_{0l}, \{\upsilon_*^h\}_{h=m}^j) \\ +f(\upsilon_*^l)\sum_{j=m}^{1}(n-m+j)P_s\left(\upsilon_*^l = \upsilon_*^j\right) P^j(\upsilon_*^l; \{\upsilon_*^h\}_{h=m}^j) \end{array}\right] +$$

$$\sum\nolimits_{j=m}^{1} P_s\left(\upsilon_*^l = \upsilon_*^j\right)\left[\begin{array}{c} f_s(\upsilon_*^l)\left[\frac{m-j}{F_s(\upsilon_*^l)} - \frac{j-1}{1-F_s(\upsilon_*^l)}\right] R^j\left(\{\upsilon_*^h\}_{h=m}^j | \upsilon_*^l = \upsilon_*^j\right) + \\ (n-m+j)F^{n-m+j-1}(\upsilon_*^l)\int_{\upsilon_*^l}^{\overline{\upsilon}}\left[\Pi_{k=1}^{m-j}\left[1-F^{n-k}(\upsilon)\right]\right] f(\upsilon)\, d\upsilon \end{array}\right] -$$

$$\upsilon_*^l f(\upsilon_*^l)\sum\nolimits_{j=m}^{1}(n-m+j)P_s\left(\upsilon_*^l = \upsilon_*^j\right) P(\upsilon_*^l; \upsilon_*^l = \upsilon_*^j) = 0.$$

Hence, we can directly derive the seller l's optimal reserve price υ_*^l as in (34). ■

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