An Empirical High-Low Model and Its Applications

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Theme:

Introduce a model for analyzing highs and lows

Illustrate the potentials of the proposed high-low model

An Empirical Model of Daily Highs and Lows
http://papers.ssrn.com/sol3/papers.cfm?abstract_id=897900

A High-Low Model of Daily Stock Price Ranges
http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1336448

A Trading Strategy Based on Callable Bull/Bear Contracts

Does Volatility Help Predict Exchange Rates?

The basic variables are

1. s_{t_i}, the i-th price observed in the t-th period

2. s_t^H, the highest price in the t-th period; $MAX_{t_i \in t}\{s_{t_i}\}$

3. s_t^L, the lowest price in the t-th period; $MIN_{t_i \in t}\{s_{t_i}\}$

Two derived variables

1. $R_t = s_t^H - s_t^L$, the range

2. $s_t^A = (s_t^H + s_t^L)/2$, the period average

Remarks:

1. For financial prices, s_{t_i}, s_t^H, s_t^L, and s_t^A are typically I(1)

2. R_t is a measure of volatility; typically I(0)

An Empirical Specification

$$\Delta \mathbf{X}_t = \mu + \Sigma_{i=1}^{p} \Gamma_i \Delta \mathbf{X}_{t-i} + \alpha R_{t-1} + \varepsilon_t$$

Δ is the differencing operator

$\mathbf{X}_t \equiv (s_t^H, s_t^L)'$

μ: 2x1 intercept vector

Γ_i: 2x2 coefficient matrix

α: 2x1 vector

$\varepsilon_t \sim$ IID(0, Σ)

Remarks:

1. A VEC representation of $\mathbf{X}_t$ with R_t as the cointegrating vector
2. An efficient specification under the cointegration assumption
3. Possible variants: nonlinear, Markov switching, threshold, long memory, …

Consider $\Lambda\Delta\mathbf{X}_t = \Lambda\mu + \Sigma_{i=1}^{p}\Lambda\Gamma_i\Delta\mathbf{X}_{t-i} + \Lambda\alpha R_{t-1} + \Lambda\varepsilon_t;\ \Lambda = \begin{pmatrix} 1/2 & 1/2 \\ 1 & -1 \end{pmatrix}$

Two Derived Specifications:

$$\Delta s_t^A = c_1 + e_1 R_{t-1} + \Sigma_{i=1}^{p}(a_i \Delta s_{t-i}^H + b_i \Delta s_{t-i}^L) + v_t,$$

$$R_t = c_2 + e_2 R_{t-1} + \Sigma_{i=1}^{p}(f_i \Delta s_{t-i}^H + g_i \Delta s_{t-i}^L) + w_t,$$

where $c_1 = (\mu_1 + \mu_2)/2,\ c_2 = \mu_1 - \mu_2,\ e_1 = (\alpha_1 + \alpha_2)/2,\ e_2 = 1 + \alpha_1 - \alpha_2$

$a_i = (\gamma_{11,i} + \gamma_{21,i})/2,\ b_i = (\gamma_{12,i} + \gamma_{22,i})/2,\ f_i = \gamma_{11,i} - \gamma_{21,i},\ g_i = \gamma_{12,i} - \gamma_{22,i}$

$v_t = (\varepsilon_{1t} + \varepsilon_{2t})/2,\ w_t = \varepsilon_{1t} - \varepsilon_{2t}$

μ_i is the i-th element of μ, α_i is the i-th element α

$\gamma_{jk,i}$ is the *jk*-th element of Γ_i, ε_{it} is the i-th element of ε_t

Remarks:

1. An agnostic model for return and risk
2. The VECM requires $\alpha_1 < 0$ and $\alpha_2 > 0$. Thus, e_1 can be +ve or –ve
3. s_t^H and s_t^L; excess demand is changing its direction; turning points, not captured by the range or the return variables
4. $\Delta s_t^A = c_1 + e_1 R_{t-1} + \Sigma_{i=1}^p a_i \Delta s_{t-i}^A + \zeta_t$; $\zeta_t = \Sigma_{i=1}^p (b_i - a_i) \Delta s_{t-i}^L + v_t$
5. $R_t = c_2 + e_2 R_{t-1} + \Sigma_{i=1}^p f_i \Delta R_{t-i} + u_t$; $u_t = \Sigma_{i=1}^p (f_i + g_i) \Delta s_{t-i}^L + w_t$

   A univarite model for R_t?
6. Asymmetric high and the low effects on average returns and ranges

Illustration I – The three major US stock indexes

An Empirical Model of Daily Highs and Lows
http://papers.ssrn.com/sol3/papers.cfm?abstract_id=897900
International Journal of Finance and Economics, 2007

Dow Jones Industrial index, NASDAQ index, S&P 500 index; 1990 to 2004

1. s_t^H and s_t^L are I(1), cointegrated; R_t is I(0), ECT

2. $\Delta \mathbf{X}_t = \mu + \Sigma_{i=1}^{p} \Gamma_i \Delta \mathbf{X}_{t-i} + \alpha R_{t-1} + \xi D_t + \varepsilon_t$

   $\alpha_1 < 0$; $\alpha_2 > 0$

   $\gamma_{11,i}$, $\gamma_{22,i} < 0$, $\gamma_{12,i}$, $\gamma_{21,i} < 0$

   Adjusted R-squares estimates: 7.9% to 17.6%

3. $\Delta \mathbf{X}_t = \mu + \Sigma_{i=1}^{p} \Gamma_i \Delta \mathbf{X}_{t-i} + \alpha R_{t-1} + \Sigma_{i=1}^{q} \Lambda_i \Delta Y_{t-i} + \Sigma_{i=1}^{r} \theta_i CO_{t-i} + \varepsilon_t$

ΔY_{t-i} includes ΔO_{t-i} and ΔC_{t-i}; CO_{t-i} is $C_{t-i} - O_{t-i}$

Adjusted R-squares estimates: 37.6% to 48.9%

4. $\Delta \mathbf{X}_t = \mu + \Sigma_{i=1}^{p} \Gamma_i \Delta \mathbf{X}_{t-i} + \alpha R_{t-1} + \Sigma_{i=0}^{s} \delta_i V_{t-i} + \varepsilon_t$

5. $\Delta \mathbf{X}_t = \mu + \Sigma_{i=1}^{p} \Gamma_i \Delta \mathbf{X}_{t-i} + \alpha R_{t-1} + \Sigma_{i=1}^{q} \Lambda_i \Delta Y_{t-i} + \Sigma_{i=1}^{r} \theta_i CO_{t-i} + \Sigma_{i=0}^{s} \delta_i V_{t-i} + \varepsilon_t$

6. Impulse responses

7. Epilogue: in-sample, alternative VECM specification (?)

Illustration II – Forecasting Ranges

A High-Low Model of Daily Stock Price Ranges
http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1336448
Journal of Forecasting, 2009

British FTSE 100, French CAC 40, German DAX 30, Nikkei 225, Korean KOSPI, US DJIA, US NASD, Taiwanese TSEC Weighted index

Estimation – Jan 3, 91 to Jan 15, 03; Forecast – Jan 16, 03 to Jun 1, 04

1. $\Delta \mathbf{X}_t = \mu + \Sigma_{i=1}^{p} \Gamma_i \Delta \mathbf{X}_{t-i} + \alpha R_{t-1} + \varepsilon_t$, it works

2. VECM forecasts – $\hat{H}_{t+h}$, $\hat{L}_{t+h}$; then $\hat{R}_{t+h}$

$$\Delta \hat{\mathbf{X}}_{t+h} = \mu + \sum_{i=1}^{p} \Gamma_i \Delta \hat{\mathbf{X}}_{t+h-i} + \alpha \hat{R}_{t+h-1}.$$

$\Delta \hat{\mathbf{X}}_{t+h-i} = \Delta \mathbf{X}_{t+h-i}$ if h-$i \le 0$; $\hat{R}_{t+h-1} = R_{t+h-1}$ if h-$1 \le 0$

$\hat{R}_{t+h,SV}$ – no estimate update; $\hat{R}_{t+h,RV}$ – estimates recursively updated

3. Competing Forecasts

$\hat{R}_{t+h,A1}$ – ARMA specifications of the ΔH_t and ΔL_t

$\hat{R}_{t+h,A2}$ – ARMA specification of the range

4. Forecast comparison

Mean-squared forecast error measure

Direction of change

Modified Diebold-Mariano statistic

5. Decomposition of Forecast Error Variance

$$\hat{R}_{t+h} - R_{t+h} = (\hat{H}_{t+h} - H_{t+h}) - (\hat{L}_{t+h} - L_{t+h})$$

$$\mathrm{V}(\hat{R}_{t+h} - R_{t+h}) = \mathrm{V}(\hat{H}_{t+h} - H_{t+h}) + \mathrm{V}(\hat{L}_{t+h} - L_{t+h}) - 2\mathrm{COV}(\hat{H}_{t+h} - H_{t+h}, \hat{L}_{t+h} - L_{t+h})$$

6. Application - implied volatility

European FTSE and DJI 1-mth and 3-mths calls and puts options contracts

$[\hat{R}^2_{t+h,j}/(4ln2)]^{1/2}$; $\hat{V}_{t+h,j} = [365\hat{R}^2_{t+h,j}/(4ln2)]^{1/2}$

Repeat (4)

7. The VECM-based range forecast does not always dominate the simple ARMA range model.

Forecast rankings depend on evaluation criteria and the variable being forecasted.

For instance, if a forecast is a good predictor of range, it may not be a good predictor of implied volatility.

Putting all these together, the in-sample results are more supportive of the VECM specification than the out-of-sample results.

Illustration III – Callable Bull/Bear Contracts

A Trading Strategy Based on Callable Bull/Bear Contracts

Callable Bull/Bear Contract (CBBC) – barrier options contract

Defining property – the contract becomes worthless once a pre-determined trigger price, the call price of the contract, is touched.

Example:

For a callable bull contract, the call price, which is the trigger price, is set either at or above the strike price. The Mandatory Call Event (MCE) occurs when the price of the underlying asset reaches the call price. When that happens, trading of the contract is terminated, the contract itself is called, and the holder of the contract is compensated according to pre-assigned terms.

A trading strategy –

Step 1: Construct forecasts of daily highs and lows for the underlying asset.

Step 2: A buy signal is generated when the forecasted daily high is no less than the call price of a callable bear contract.

Step 3: Buy the underlying asset when the buy signal is observed for *m* consecutive days.

Step 4: Cover the long position when either the buy signal disappears for *m* consecutive days, the MCE occurs, or the CBBC matures.

Step 5: If the position is closed before the MCE and contract maturity, then repeat Steps 2 to 4.

In essence, the strategy uses the call price and the MCE to define the entry and the exit point.

Remarks:

1. A sell signal is generated when the forecasted daily low is no larger than the call price of a callable bull contract. Steps 3 and 4 can be modified to accommodate a short-sell of the underlying asset.

2. The VECM high-low model can be used to generate high and low forecasts.

3. For practical reasons, we buy or sell instruments that track the performance of the underlying asset instead of the CBBCs themselves.

4. m is a choice parameter. A small m value can introduce substantial noise to the trading signal and generate too many trades. On the other hand, too many profitable trading opportunities will be forgone if m is too large.

Data

- CBBCs traded in the HK Exchange
- Underlying asset, HIS; June 2006 to December 2008

Results

- Decent trading returns on average
- Returns vary quite substantially across individual trades
- Trading returns are associated with
  - Volatility during a contract’s lifespan
  - To a lesser extent, with volatility in the pre-issuance period
  - An issuer’s relative issuing frequency (average group)

Illustration IV – The Economic Value of Forecasting Exchange Rates

Does Volatility Help Predict Exchange Rates?

Exchange rate forecasting

statistical evaluation Vs economic evaluation

a random walk benchmark

1. $$\Delta \mathbf{X}_t = \mu + \Sigma_{i=1}^{p} \Gamma_i \Delta \mathbf{X}_{t-i} + \alpha R_{t-1} + \varepsilon_t$$

$$\Delta s_t^A = c_1 + e_1 R_{t-1} + \Sigma_{i=1}^{p} (a_i \Delta s_{t-i}^H + b_i \Delta s_{t-i}^L) + v_t$$

Build-in risk-return relationship

A relatively simple and intuitive way to generate forecasts of *both* exchange rates and their conditional volatility

2. Out-of-sample forecasting: A statistical assessment

3. Economic Assessment

Allocation between safe domestic and foreign bonds

Mean-variance framework

$$\omega_t = \frac{E_t\left(i_t^* + \Delta s_{t+1}^A - i_t\right)}{\lambda \operatorname{var}_t\left(i_t^* + \Delta s_{t+1}^A - i_t\right)} = \frac{\left(i_t^* - i_t\right) + E_t\left(\Delta s_{t+1}^A\right)}{\lambda \operatorname{var}_t\left(\Delta s_{t+1}^A\right)}$$

4. Performance comparison

$$SR = \frac{E\left(r_{t+1} - i_t\right)}{\operatorname{var}\left(r_{t+1}\right)}$$

The (maximum) performance fee that can be deduced from the proposed strategy's returns such the portfolio strategies based on the RW model and the proposed model offer the same average utility

$$\bar{U} = \frac{W_0}{T} \sum_{t=0}^{T-1} \left\{ 1 + r_{t+1} - \frac{\lambda}{2(1+\lambda)} (1 + r_{t+1})^2 \right\}$$

$$\sum_{t=0}^{T-1} \left\{ \left[\left(1 + r_{t+1}^{VECM} \right) - \Phi \right] - \frac{\lambda}{2(1+\lambda)} \left[\left(1 + r_{t+1}^{VECM} \right) - \Phi \right]^2 \right\}$$

$$= \sum_{t=0}^{T-1} \left\{ \left(1 + r_{t+1}^{RW} \right) - \frac{\lambda}{2(1+\lambda)} \left(1 + r_{t+1}^{RW} \right)^2 \right\}$$

The transaction costs equal a fixed proportion of the value traded in each bond. The break-even transaction cost is the level of costs that render the portfolio return zero.

5. An Alternative benchmark

$$\left(r_t^{VECM} - i_t\right) = \varphi_0 + \sum_{q=1}^{4} \varphi_q F_t^q + e_t$$

The four currency strategies: carry trade, trend following, value, and volatility trading.

Alpha returns φ_0's are all positive and mostly statistically significant.

Positive association with the trend-following factor

No consistent association with three other factors

6. A Caveat

Working (1960)

Illustration V – Range and EVT

Work in Progress

Could we extend the analysis to, say, the 95% quantile?

How does it compare with procedures from EVT?